8. [11 points] In this problem, we consider the parametric curve given by

$$x = f(t) \qquad\qquad y = g(t)$$

for all t, where f and g are twice-differentiable functions. Some values of f and g and their derivatives are given in the tables below.

t	1	2	3	4	5
$f(t)$	-3	-4	-3	-1	1
$g(t)$	5	2	-2	-4	-1

t	1	2	3	4	5
$f'(t)$	-2	0	1	3	1
$g'(t)$	-4	-2	-1	0	2

a. [1 point] In the space provided, write an integral that gives the arc length of the parametric curve from $t = 1$ to $t = 5$.

Arc length = $\int_1^5 \sqrt{(f'(t))^2 + (g'(t))^2}\,dt$

b. [3 points] Use a midpoint sum with as many subdivisions as possible to estimate your integral from part **a**. Write out all the terms in your sum, and do **not** simplify.

Solution: The midpoint sum is $2(\sqrt{0^2 + (-2)^2} + \sqrt{3^2 + 0^2})$.

c. [3 points] Find the Cartesian equation for the tangent line to the parametric curve in the xy-plane at $t = 1$.

Solution: In point-slope form, the tangent line is given by $y - 5 = 2(x + 3)$.

d. [2 points] Consider the tangent lines to the parametric curve at the t-values $t = 1, 2, 3, 4, 5$. Are any of these lines **perpendicular** to each other? If so, list any **two** t-values for which the tangent lines are perpendicular. If not, write "NO."

Solution: The tangent lines corresponding to $t = 2$ and $t = 4$ are perpendicular.

e. [2 points] As t ranges from 1 to 5, the corresponding part of the parametric curve intersects the line $y = x$ exactly once. Which interval contains the t-value for which the curve intersects the line $y = x$? Circle your answer. You do not need to show any work.

$(1, 2)$ $\qquad$ $(2, 3)$ $\qquad$ $\boxed{(3, 4)}$ $\qquad$ $(4, 5)$