

10. [12 points]

a. [6 points] Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{n \cdot 3^n}.$$

Show every step of any calculations and fully justify your answer with careful reasoning. Write your final answers on the answer blanks provided.

**Answer:** Radius of Convergence: \_\_\_\_\_

Interval of Convergence: \_\_\_\_\_

b. [3 points] The Maclaurin series for a function  $f(x)$  is the power series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{n \cdot 3^n}$ .

The function  $f(x)$  is closely related to one of the functions that appears on the formula sheet on the last page of this exam. Find a formula for  $f(x)$  in closed form (i.e. without sigma notation or ellipses ( $\dots$ )).

**Answer:**  $f(x) =$  \_\_\_\_\_

c. [3 points] Suppose the Taylor series about  $x = 0$  for a function  $g(x)$  is  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} x^{3n}$ .

Let  $h(x) = g(\frac{x}{2})$ . Find  $h^{(15)}(0)$ .

**Answer:**  $h^{(15)}(0) =$  \_\_\_\_\_