11. [14 points] An auto manufacturer is testing the braking capability of one of its hybrid-electric vehicles. At regular time intervals during the experiment, the auto engineers measure the speed and the position of the car along the test track. Let $t$ be the number of seconds after the car begins braking.

Let $v(t)$ be the car’s speed at time $t$, in meters per second, and let $p(t) = \int_0^t v(s)\, ds$.

The auto engineers are most interested in the time period $0 \leq t \leq 40$, when the car’s acceleration is always negative but increasing.

The velocity measurements taken during this time period are given in the table below.

<table>
<thead>
<tr>
<th>$t$ (seconds)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(t)$ (m/s)</td>
<td>111</td>
<td>60</td>
<td>25</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

a. [3 points] Consider the four approximations of the definite integral $\int_0^{40} v(t)\, dt$ given by \text{RIGHT}(4), \text{LEFT}(4), \text{TRAP}(4), and MID(4). Rank these five quantities in order from least to greatest by filling in the blanks below with the options I–V.

\[
\text{I. } \int_0^{40} v(t)\, dt \quad \text{II. RIGHT}(4) \quad \text{III. LEFT}(4) \\
\text{IV. TRAP}(4) \quad \text{V. MID}(4)
\]

b. [3 points] Write out all the terms of the LEFT(4) approximation of $\int_0^{40} v(t)\, dt$.

c. [4 points] Let $h(x)$ be the gasoline fuel efficiency of the test vehicle, in liters per hectokilometer (i.e. liters per 100 km) when the car is traveling at a speed of $x$ m/s.

i. Suppose a formula for $h$ is given by $h(x) = 2.3 + 0.097x$.

Compute the value of $\int_0^{40} h'(v(t)) \cdot v'(t)\, dt$.

\[
\text{Answer: } \int_0^{40} h'(v(t)) \cdot v'(t)\, dt = \]

\[\text{__________________________}\]
ii. Let

\[ K = \int_0^{40} h'(v(t)) \cdot v'(t) \, dt \]

(Note that \( K \) is the value you computed in part c(i).)
Circle the phrase below that best completes the practical interpretation of \( K \) that begins “During the last 40 seconds of the experiment…”

I. the vehicle consumes \( |K| \) liters of fuel per hectokilometer.

II. the rate of change of the vehicle’s fuel efficiency is \( K \) liters per hectokilometer per second.

III. the vehicle consumes \( |K| \) liters of fuel.

IV. the total change in the rate of change of fuel in the vehicle’s gas tank is \( 1/K \) liters per second.

V. the total change in the vehicle’s fuel efficiency is \( K \) liters per hectokilometers.

d. [4 points] The energy density of the car’s battery is a function of time, \( E(t) \), which can be multiplied by the car’s position function \( p(t) \) in order to compute the battery’s charge. Suppose that \( E(0) = 1, E(40) = 0.89, E'(0) = -0.0028, \) and \( E'(40) = -0.025 \).
Use your answer to part b above to estimate the value of

\[ \int_0^{40} (v(t)E(t) + p(t)E'(t)) \, dt. \]

Hint: What is \( p'(t) \)?