11. [14 points] An auto manufacturer is testing the braking capability of one of its hybrid-electric vehicles. At regular time intervals during the experiment, the auto engineers measure the speed and the position of the car along the test track.
Let t be the number of seconds after the car begins braking.

Let $v(t)$ be the car's speed at time t, in meters per second, and let $p(t) = \int_0^t v(s)\,ds$.

The auto engineers are most interested in the time period $0 \le t \le 40$, when the car's acceleration is always negative but increasing.
The velocity measurements taken during this time period are given in the table below.

t (seconds)	0	10	20	30	40
$v(t)$ (m/s)	111	60	25	5	0

a. [3 points] Consider the four approximations of the definite integral $\int_0^{40} v(t)\,dt$ given by RIGHT(4), LEFT(4), TRAP(4), and MID(4). Rank these five quantities in order from least to greatest by filling in the blanks below with the options I–V.

I. $\int_0^{40} v(t)\,dt$ II. RIGHT(4) III. LEFT(4) IV. TRAP(4) V. MID(4)

_____ < _____ < _____ < _____ < _____

b. [3 points] Write out all the terms of the LEFT(4) approximation of $\int_0^{40} v(t)\,dt$.

c. [4 points] Let $h(x)$ be the gasoline fuel efficiency of the test vehicle, in liters per hectokilometer (i.e. liters per 100 km) when the car is traveling at a speed of x m/s.

i. Suppose a formula for h is given by $h(x) = 2.3 + 0.097x$.
Compute the value of $\int_0^{40} h'(v(t)) \cdot v'(t)\,dt$.

Answer: $\int_0^{40} h'(v(t)) \cdot v'(t)\,dt =$ _____

This is a continuation of the problem from the previous page.

ii. Let

$$K = \int_0^{40} h'(v(t)) \cdot v'(t)\, dt$$

(Note that K is the value you computed in part c(i).)
Circle the phrase below that best completes the practical interpretation of K that begins "*During the last 40 seconds of the experiment...*"

I. the vehicle consumes $|K|$ liters of fuel per hectokilometer.

II. the rate of change of the vehicle's fuel efficiency is K liters per hectokilometer per second.

III. the vehicle consumes $|K|$ liters of fuel.

IV. the total change in the rate of change of fuel in the vehicle's gas tank is $1/K$ liters per second.

V. the total change in the vehicle's fuel efficiency is K liters per hectokilometers.

d. [4 points] The energy density of the car's battery is a function of time, $E(t)$, which can be multiplied by the car's position function $p(t)$ in order to compute the battery's charge. Suppose that $E(0) = 1$, $E(40) = 0.89$, $E'(0) = -0.0028$, and $E'(40) = -0.025$. Use your answer to part **b** above to estimate the value of

$$\int_0^{40} \big(v(t)E(t) + p(t)E'(t)\big)\, dt.$$

Hint: What is $p'(t)$?