

7. [9 points] The “Witch of Maria Agnesi” is the family of curves described by the parametric equations

$$x = 2at \quad \text{and} \quad y = \frac{2a}{1+t^2}$$

for all t , where a is a positive constant.

- a. [4 points] Consider a “Witch curve” as defined above.

- i. Find a formula for $\frac{dy}{dx}$ in terms of t and/or a .

Answer: $\frac{dy}{dx}$ _____

- ii. Find a formula for the tangent line to the “Witch curve” at the point where $t = 1$. Your answer might involve the constant a but should **not** involve t .

Answer: $y =$ _____

- b. [5 points] The total area in the first quadrant that is bounded between a “Witch curve” and the x -axis is represented by the improper integral $\int_0^{\infty} \frac{8a^3}{x^2 + 4a^2} dx$.

Determine whether this improper integral converges or diverges.

- If the integral converges, circle “converges”, find its exact value (i.e. no decimal approximations), and write the exact value on the answer blank provided. (The exact value may involve the constant a .)
- If the integral diverges, circle “diverges” and carefully justify your answer.

In either case, **you must show all your work carefully using correct notation.** Any direct evaluation of integrals must be done **without using a calculator.**

Converges to _____

Diverges

Hint: Note that $\frac{d}{dx} \left(4a^2 \arctan \left(\frac{x}{2a} \right) \right) = \frac{8a^3}{x^2 + 4a^2}$.