11. [14 points] An auto manufacturer is testing the braking capability of one of its hybrid-electric vehicles. At regular time intervals during the experiment, the auto engineers measure the speed and the position of the car along the test track.

Let \( t \) be the number of seconds after the car begins braking.

Let \( v(t) \) be the car’s speed at time \( t \), in meters per second, and let \( p(t) = \int_0^t v(s) \, ds \).

The auto engineers are most interested in the time period \( 0 \leq t \leq 40 \), when the car’s acceleration is always negative but increasing. The velocity measurements taken during this time period are given in the table below.

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) ) (m/s)</td>
<td>111</td>
<td>60</td>
<td>25</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

a. [3 points] Consider the four approximations of the definite integral \( \int_0^{40} v(t) \, dt \) given by RIGHT(4), LEFT(4), TRAP(4), and MID(4). Rank these five quantities in order from least to greatest by filling in the blanks below with the options I–V.

\[
\text{I. } \int_0^{40} v(t) \, dt \quad \text{II. RIGHT(4)} \quad \text{III. LEFT(4)} \quad \text{IV. TRAP(4)} \quad \text{V. MID(4)}
\]

\[
\text{II} < \text{V} < \text{I} < \text{IV} < \text{III}
\]

b. [3 points] Write out all the terms of the LEFT(4) approximation of \( \int_0^{40} v(t) \, dt \).

\[
\text{LEFT(4)} = \Delta t \left[ v_0 + v_1 + v_2 + v_3 + v_4 \right] = 10 \left[ 111 + 60 + 25 + 5 \right]
\]

\[
= 10 \left[ 111 + 60 + 25 + 5 \right] = 10 \cdot 201 = 2010
\]

\[
\left. \begin{array}{c}
\text{LEFT(4)} = \Delta t \left[ v_0 + v_1 + v_2 + v_3 + v_4 \right] \\
= 10 \left[ 111 + 60 + 25 + 5 \right]
\end{array} \right\} = 2010
\]

\[
\int_0^4 h'(v(t)) \cdot v'(t) \, dt = -10.767
\]

\[
\text{Let } w = v(t) \\
\frac{dw}{dt} = v'(t) \\
\int_0^4 w' \, dw = \frac{1}{2} w^2 \bigg|_0^4 = 2 \cdot 4^2 = 32 \]

\[
h(4) - h(0) = 2.3 - (2.3 + 10.767) = -10.767
\]

\[
\int_0^4 h'(v(t)) \cdot v'(t) \, dt = -10.767
\]
This is a continuation of the problem from the previous page.

ii. Let

\[ K = \int_{0}^{40} h'(v(t)) \cdot v'(t) \, dt \]

(Note that \( K \) is the value you computed in part c(i).)

Circle the phrase below that best completes the practical interpretation of \( K \) that begins “During the last 40 seconds of the experiment...”

I. the vehicle consumes \(|K|\) liters of fuel per hectokilometer.

II. the rate of change of the vehicle’s fuel efficiency is \( K \) liters per hectokilometer per second.

III. the vehicle consumes \(|K|\) liters of fuel.

IV. the total change in the rate of change of fuel in the vehicle’s gas tank is \( 1/K \) liters per second.

V. the total change in the vehicle’s fuel efficiency is \( K \) liters per hectokilometers.

\[ \text{d. [4 points]} \]

The energy density of the car’s battery is a function of time, \( E(t) \), which can be multiplied by the car’s position function \( p(t) \) in order to compute the battery’s charge. Suppose that \( E(0) = 1, E(40) = 0.89, E'(0) = -0.0028, \) and \( E'(40) = -0.025 \).

Use your answer to part b above to estimate the value of

\[ \int_{0}^{40} (v(t)E(t) + p(t)E'(t)) \, dt. \]

Hint: What is \( p'(t) \)?