11. [14 points] An auto manufacturer is testing the braking capability of one of its hybrid-electric vehicles. At regular time intervals during the experiment, the auto engineers measure the speed and the position of the car along the test track.
Let t be the number of seconds after the car begins braking.

Let $v(t)$ be the car's speed at time t, in meters per second, and let $p(t) = \int_0^t v(s)\, ds$.

The auto engineers are most interested in the time period $0 \le t \le 40$, when the car's acceleration is always negative but increasing.
The velocity measurements taken during this time period are given in the table below.

t (seconds)	0	10	20	30	40
$v(t)$ (m/s)	111	60	25	5	0

a. [3 points] Consider the four approximations of the definite integral $\int_0^{40} v(t)\, dt$ given by RIGHT(4), LEFT(4), TRAP(4), and MID(4). Rank these five quantities in order from least to greatest by filling in the blanks below with the options I–V.

v' negative but increasing $\Rightarrow$ v dec, conc up

I. $\int_0^{40} v(t)\, dt$ II. RIGHT(4) III. LEFT(4)

IV. TRAP(4) V. MID(4)

II < V < I < IV < III

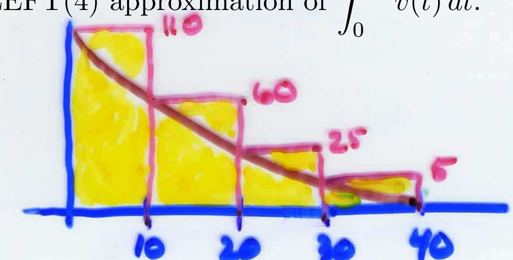
RIGHT < ∫ since dec; MID < ∫ since V conc up; ∫; TRAP > ∫ since V conc up; LEFT > ∫ since dec

b. [3 points] Write out all the terms of the LEFT(4) approximation of $\int_0^{40} v(t)\, dt$.

$\Delta t = \frac{40-0}{4} = 10$

LEFT(4) $= \Delta t\,[v_0 + v_1 + v_2 + v_3]$

$\boxed{= 10\,[111 + 60 + 25 + 5]}$



c. [4 points] Let $h(x)$ be the gasoline fuel efficiency of the test vehicle, in liters per hectokilometer (i.e. liters per 100 km) when the car is traveling at a speed of x m/s.

i. Suppose a formula for h is given by $h(x) = 2.3 + 0.097x$.
Compute the value of $\int_0^{40} h'(v(t)) \cdot v'(t)\, dt$.

Let $w = v(t)$
$dw = v'(t)\,dt$
$t = 0 \Rightarrow w = 111$
$t = 40 \Rightarrow w = 0$

$= \int_{111}^{0} h'(w)\, dw = h(w)\Big|_{111}^{0}$

$= h(0) - h(111) = 2.3 - (2.3 + 10.767)$

Answer: $\int_0^{40} h'(v(t)) \cdot v'(t)\, dt =$ $\boxed{-10.767}$

This is a continuation of the problem from the previous page.

ii. Let

$$K = \int_0^{40} h'(v(t)) \cdot v'(t)\, dt$$

(Note that K is the value you computed in part c(i).)
Circle the phrase below that best completes the practical interpretation of K that begins "*During the last 40 seconds of the experiment...*"

I. the vehicle consumes $|K|$ liters of fuel per hectokilometer.

II. the rate of change of the vehicle's fuel efficiency is K liters per hectokilometer per second.

III. the vehicle consumes $|K|$ liters of fuel.

IV. the total change in the rate of change of fuel in the vehicle's gas tank is $1/K$ liters per second.

V. the total change in the vehicle's fuel efficiency is K liters per hectokilometers.

d. [4 points] The energy density of the car's battery is a function of time, $E(t)$, which can be multiplied by the car's position function $p(t)$ in order to compute the battery's charge. Suppose that $E(0) = 1$, $E(40) = 0.89$, $E'(0) = -0.0028$, and $E'(40) = -0.025$. Use your answer to part **b** above to estimate the value of

$$\int_0^{40} \big(v(t)E(t) + p(t)E'(t)\big)\, dt.$$

$p(t) = \int_0^t v(s)\,ds$

so $p'(t) = v(t)$

Hint: What is $p'(t)$?

we recognize the product rule:

$$\int_0^{40} (v(t)E(t) + p(t)E'(t))\,dt$$

$$= \int_0^{40} (p'(t)E(t) + p(t)E'(t))\,dt = \int_0^{40} \frac{d}{dt}\big[p(t)E(t)\big]\,dt$$

$= p(t)E(t)\Big|_0^{40} = p(40)E(40) - p(0)E(0)$. $p(0) = 0$ and $E(40) = .89$, so that's $.89\,p(40) - 0 = .89\int_0^{40} v(s)\,ds$

$\approx .89$ (estimate from part b) $= .89(2010) = \boxed{1788.9}$