12. [6 points] For each of the questions below, circle all of the available correct answers.
Circle "NONE OF THESE" if none of the available choices are correct.
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

a. [2 points] Which of the following converge to the number $\int_0^{\pi/2} e^{-x^2}\,dx$?

i. The sequence LEFT(n) of approximations of $\int_0^{\pi/2} e^{-x^2}\,dx$, for $n \geq 1$.

ii. The sequence $\int_{1/n}^{\pi/2} e^{-x^2}\,dx$ for $n \geq 1$

iii. The series $\sum_{n=0}^{\infty} \frac{(-1)^n(\pi^{2n+1})}{(2n+1)(2^{2n+1})(n!)}$

iv. NONE OF THESE

Let $F(x) = \sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\,n!}$. Then $F'(x) = \sum \frac{(-1)^n x^{2n}}{n!} = \sum \frac{(-x^2)^n}{n!} = e^{-x^2}$. So $\int_0^{\pi/2} e^{-x^2} = F(\frac{\pi}{2}) - F(0)$.

b. [2 points] Consider the sequence $a_n = \frac{1}{\ln(n)}$, $n \geq 2$. Which of the following statements are true?

i. $\lim_{n\to\infty} a_n = 0$.

$\ln(n) \to \infty$ so $\frac{1}{\ln(n)} \to 0$

ii. The series $\sum_{n=2}^{\infty} a_n$ converges.

$\ln(n) < n \Rightarrow \frac{1}{\ln(n)} > \frac{1}{n}$.

iii. The series $\sum_{n=2}^{\infty} a_n$ diverges.

Since $\sum \frac{1}{n}$ diverges, $\sum \frac{1}{\ln(n)}$ diverges by comparison.

iv. The series $\sum_{n=2}^{\infty} (-1)^n a_n$ converges.

v. NONE OF THESE

conv by alt. series test

c. [2 points] Which of the following series are conditionally convergent?

i. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ conv by AST, but $\sum \frac{1}{n}$ div by integral test

ii. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ absolutely convergent by p-test (or integral test)

iii. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$ abs convergent by ratio test (geometric series $\to \frac{-1/3}{1+1/3} = -\frac{1}{4}$)

iv. NONE OF THESE