2. [8 points] For this problem, consider the family of polar curves described for each positive integer $n \geq 1$ by

$$r = \frac{\cos(2n\theta)}{n}$$

for $0 \leq \theta \leq 2\pi$.

a. [2 points] Consider the polar curve described by $r = \cos(2\theta)$ for $0 \leq \theta \leq 2\pi$. (Note that this is the case of $n = 1$.) Find all values of θ between 0 and 2π for which the curve $r = \cos(2\theta)$ passes through the origin.

At origin $\iff r=0 \iff \cos(2\theta)=0$

$\iff 2\theta = m\pi + \frac{\pi}{2}$ for some integer m

$\iff \theta = m\frac{\pi}{2} + \frac{\pi}{4}$ for some integer m

Answer: $\theta = \boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

b. [3 points] For $n \geq 1$, find all x-intercepts of the polar curve $r = \dfrac{\cos(2n\theta)}{n}$. Your answer(s) may involve n.

x-intercept $\iff 0 = y = r\sin\theta \iff r=0$ or $\sin\theta = 0$.
r can be 0, as when $\theta = \pi/4n$. And $\sin\theta = 0$ when $\theta = m\pi$ for some integer m. In that case, $r = \frac{\cos(2nm\pi)}{n} = \frac{1}{n}$ and $\cos\theta = \begin{cases} 1 & \text{if } m \text{ even} \\ -1 & \text{if } m \text{ odd.} \end{cases}$

So $x = \pm\frac{1}{n}$.

Answer: $x = \boxed{-\frac{1}{n}, 0, \frac{1}{n}}$

c. [3 points] For $n \geq 1$, let A_n be the arclength of the polar curve $r = \dfrac{\cos(2n\theta)}{n}$ for $0 \leq \theta \leq 2\pi$. Write, but do not evaluate, an expression involving one or more integrals that gives the value of A_n.

$$A_n = \int_0^{2\pi} \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$$

where $f(\theta) = \frac{1}{n}\cos(2n\theta)$

$f'(\theta) = -2\sin(2n\theta)$

Answer: $A_n = \boxed{\int_0^{2\pi} \sqrt{\frac{1}{n^2}\cos^2(2n\theta) + 4\sin^2(2n\theta)}\, d\theta}$