3. [11 points] Show your work, but no explanation is necessary. For parts **a**, **c**, and **d**, be sure to pay close attention to whether the question is asking you for a median or a mean.

a. [3 points] Compute the **median** value of a quantity that has cumulative distribution function given by

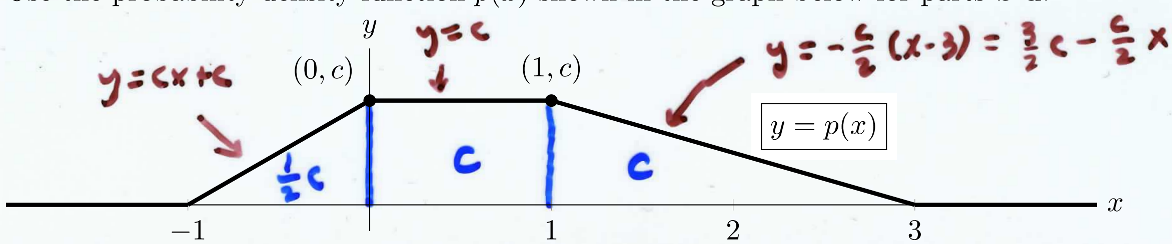
$$\text{CDF:} \quad F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-(x/r)^k} & \text{if } x \geq 0 \end{cases}$$

Here r and k are constants, and your answer may involve one or both of these constants.

$$\frac{1}{2} = F(x) = 1 - e^{-(x/r)^k} \Rightarrow e^{-(x/r)^k} = \frac{1}{2} \Rightarrow -\left(\frac{x}{r}\right)^k = \ln\frac{1}{2} = -\ln 2$$

Answer: median = $\boxed{r(\ln 2)^{1/k}}$

Use the probability density function $p(x)$ shown in the graph below for parts **b-d**.



b. [2 points] Use the fact that the graph above shows a probability density function to find the value of the constant c.

$$1 = \text{area under pdf} = \frac{1}{2}c + c + c = \frac{5}{2}c$$

Answer: $c = \boxed{2/5}$

c. [3 points] Compute the **mean** of the quantity with probability density function shown in the graph above.

$$\text{mean} = \int_{-\infty}^{\infty} x p(x)\,dx = \int_{-1}^{0} x(cx+c) + \int_0^1 x(c) + \int_1^3 x\left(\frac{3}{2}c - \frac{c}{2}x\right)$$

$$= c\left[\int_{-1}^{0} x^2 + x + \int_0^1 x + \int_1^3 \frac{3}{2}x - \frac{1}{2}x^2\right] = \frac{2}{5}\left[-\frac{1}{6} + \frac{1}{2} + \frac{5}{3}\right]$$

Answer: mean = $\boxed{4/5}$

d. [3 points] Compute the **median** of the quantity with probability density function shown in the graph above. Guess median is between 0 and 1.

Then

$$\frac{1}{2} = \frac{1}{2}c + \int_0^m c = \frac{1}{2}c + mc \Rightarrow m = \frac{1/2 - 1/2\,c}{c} = \frac{1/2 - 1/5}{2/5}$$

Answer: median = $\boxed{3/4}$