3. [11 points] Show your work, but no explanation is necessary. For parts a, c, and d, be sure to pay close attention to whether the question is asking you for a median or a mean.

a. [3 points] Compute the median value of a quantity that has cumulative distribution function given by

\[ F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\frac{(x/r)^k}{r}} & \text{if } x \geq 0 \end{cases} \]

Here \( r \) and \( k \) are constants, and your answer may involve one or both of these constants.

\[
\frac{1}{2} = F(x) = 1 - e^{-\frac{(x/r)^k}{r}} \Rightarrow e^{-\frac{(x/r)^k}{r}} = \frac{1}{2} \Rightarrow -\left(\frac{x}{r}\right)^k = \ln \frac{1}{2} = \ln 2
\]

Answer: median = \( r \left( \frac{\ln 2}{k} \right) \)

b. [2 points] Use the fact that the graph above shows a probability density function to find the value of the constant \( c \).

\[
1 = \text{area under pdf} = \frac{1}{2}c + c + c = \frac{5}{3}c
\]

Answer: \( c = \frac{3}{\sqrt{5}} \)

c. [3 points] Compute the mean of the quantity with probability density function shown in the graph above.

\[
\text{mean} = \int_{-\infty}^{\infty} x \cdot p(x) \, dx = \int_{-1}^{0} x \cdot (c+1) + \int_{0}^{1} x \cdot (c-1) + \int_{1}^{3} x \cdot \left(\frac{3}{3}c - \frac{5}{3}x\right)
\]

\[
= c \left[ \int_{-1}^{0} x^2 + x + \int_{0}^{1} x + \int_{1}^{3} \frac{3}{3}x - \frac{5}{3}x^2 \right] = \frac{2}{3} \left[ -\frac{1}{6} + \frac{1}{2} + \frac{5}{3} \right]
\]

Answer: mean = \( \frac{4}{\sqrt{5}} \)

d. [3 points] Compute the median of the quantity with probability density function shown in the graph above.

Guess median is between 0 and 1.

Then \( \frac{1}{2} = \frac{1}{2}c + \int_{0}^{c} \frac{3}{3}c = \frac{1}{2}c + mc \Rightarrow m = \frac{\frac{1}{2} - \frac{1}{2}c}{c} = \frac{\frac{1}{2} - \frac{1}{2}}{2/\sqrt{5}} \)

Answer: median = \( \frac{3}{4} \)