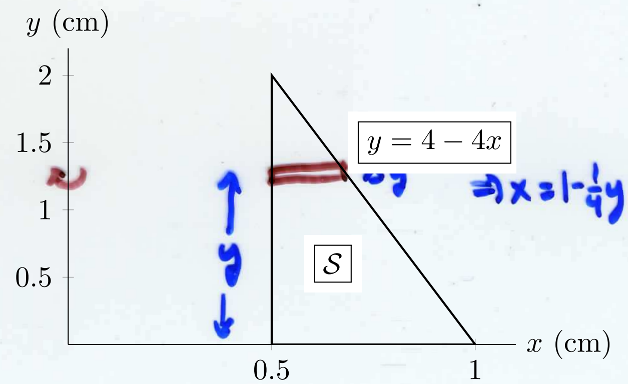
4. [4 points]

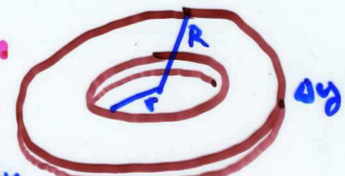
Let $\mathcal{S}$ be the region bounded by the x-axis, the line $x = 0.5$, and the line $y = 4 - 4x$. This region is shown to the right. The units on both the x- and the y-axis are centimeters. A solid is obtained by rotating the region $\mathcal{S}$ about the y-axis. The mass density of the resulting solid at each point y centimeters above the x-axis is $16y$ grams per cubic centimeter.



Write, but do **not** evaluate, an expression involving one or more integrals that gives the mass, in grams, of the resulting solid.

inner radius of slice $= r = 0.5$ cm

outer " " " $= R = 1 - \frac{1}{4}y$ cm



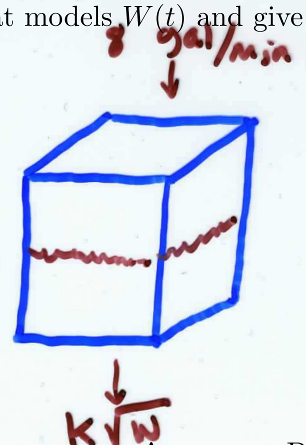
Volume " " $= \pi R^2 \Delta y - \pi r^2 \Delta y$

$= \pi [R^2 - r^2] \Delta y = \pi [(1 - \frac{1}{4}y)^2 - (.5)^2] \Delta y$ cm^3

mass $=$ Volume $\cdot$ density $= \pi ((1 - \frac{1}{4}y)^2 - (\frac{1}{2})^2) \Delta y (16y)$ g

Answer: Mass $= 16\pi \int_0^2 \left((1 - \frac{1}{4}y)^2 - (\frac{1}{2})^2\right) y \, dy$ g

5. [5 points] Prior to the start of an indoor winter carnival, the water tank for a dunking booth is being filled from a water hose at a rate of 8 gallons per minute. Unfortunately, once the tank has 50 gallons of water in it, the tank begins leaking water at a rate (in gallons per minute) that is proportional to the square root of the volume of water in the tank (in gallons) with constant of proportionality $k > 0$. Let $W = W(t)$ be the volume, in gallons, of water that is in the tank t minutes after the tank begins to leak. Write a differential equation that models $W(t)$ and give an appropriate initial condition.



Answer: Differential Equation: $\frac{dW}{dt} = 8 - k\sqrt{W}$

Initial Condition: $W(0) = 50$