6. [8 points]

Values of a function f and some of its derivatives are given in the table on the right. Use this information to answer the questions that follow.

x	0	π
$f(x)$	-6	2π
$f'(x)$	6	2
$f''(x)$	1	-3
$f'''(x)$	-1	0
$f''''(x)$	5	$-9/2$

a. [4 points] Find a formula for the Taylor polynomial of degree 4 for f about $x = \pi$.

$$P_4(x) = \sum_{n=0}^{4} \frac{f^{(n)}(\pi)}{n!}(x-\pi)^n$$

$$= f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2 + \frac{f'''(\pi)}{3!}(x-\pi)^3 + \frac{f^{IV}(\pi)}{4!}(x-\pi)^4$$

$$= 2\pi + 2(x-\pi) + \frac{-3}{2}(x-\pi)^2 + \frac{0}{6}(x-\pi)^3 + \frac{-9/2}{24}(x-\pi)^4$$

$$= \boxed{2\pi + 2(x-\pi) - \frac{3}{2}(x-\pi)^2 - \frac{3}{16}(x-\pi)^4}$$

b. [4 points] Find the first three nonzero terms of the Taylor series for $\int_0^x f(t^2)\,dt$ about $x = 0$.

Near 0, $f(x) \approx f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2$

$$= -6 + 6x + \frac{1}{2}x^2$$

So $\int_0^x f(t^2)\,dt \approx \int_0^x -6 + 6t^2 + \frac{1}{2}t^4\,dt$

$$= -6t + 2t^3 + \frac{1}{10}t^5 \Big|_0^x$$

$$= \boxed{-6x + 2x^3 - \frac{1}{10}x^5}$$