9. [12 points] For each of the questions on this page:
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

For parts **a** - **c** below, circle all of the available correct answers, and circle "NONE OF THESE" if none of the available options are correct.

a. [4 points] Suppose a_n and b_n are nonzero sequences. Functions P and Q satisfy the following: $P(x) = \sum_{n=0}^{\infty} a_n(x-1)^n$ for $-1 < x \leq 3$ and $Q(x) = \sum_{n=0}^{\infty} b_n x^n$ for $-1 \leq x \leq 1$.
Which of the following must be true?

i. The radius of convergence of the Taylor series for $P(x)$ around $x = 1$ is at least 1.

ii. $\sum_{n=1}^{\infty} \frac{b_n}{n}$ converges. iii. $\sum_{n=0}^{\infty} a_n 2^n$ diverges. iv. $\sum_{n=0}^{\infty} \frac{1}{a_n}$ diverges.

v. The Taylor series for $P(x)$ around $x = 0$ is $\sum_{n=0}^{\infty} a_n x^n$. vi. NONE OF THESE

b. [4 points] Suppose $f(x)$ is a positive, decreasing, and concave up function. Suppose further that all derivatives of $f(x)$ exist at $x = 0$. Define $F(x) = \int_0^x f(t)\,dt$.
Which of the following must be true?

i. TRAP(n) is an overestimate of $\int_0^1 F(x)\,dx$ for all positive integers n.

ii. $F(x) + F''(x)$ is an increasing function.

iii. The Taylor series for $F(x)$ and for $f(x)$ centered around $x = 0$ both have the same radius of convergence.

iv. $\int_0^1 \frac{f(x)}{F(x)}\,dx$ converges. v. $\sum_{n=1}^{\infty} f(n)$ converges. vi. NONE OF THESE

c. [4 points] Consider the differential equation $y' = (\cos(x) - \sin(y))^2$, and suppose $y = g(x)$ is the solution to this differential equation that passes through the point $(0, 0)$.
Which of the following must be true?

i. This differential equation has no equilibrium solutions. ii. $g''(0) = -2$.

iii. $y = \arcsin(\cos(x))$ is an equilibrium solution. iv. $g(x) \leq 4x$ for all $x > 0$

v. $g(x)$ is increasing. vi. NONE OF THESE