**1.** [15 points] The table below gives several values of a twice differentiable function $f$ along with its derivative $f'$ and continuous second derivative $f''$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 2.4 | 2.5 | 2.2 | 2.6 | 4.3 | 6.7 |
| $f'(x)$ | 2 | 0.7 | $-0.3$ | $-0.1$ | 1.1 | 2.2 | 2.2 |
| $f''(x)$ | $-1$ | $-1.4$ | $-0.5$ | 0.8 | 1.4 | 0.7 | $-0.7$ |

Unless otherwise stated, you do not have to show work, but work shown might be considered for partial credit.

**a.** [3 points] Find the value of $\int_1^4 xf''(x)\,dx$.

Let $u = x$ $\quad v' = f''(x)$

$u' = 1$ $\quad v = f'(x)$

$$\int xf''(x)dx = \int uv'dx = uv - \int u'v\,dx = xf'(x) - \int (1)f'(x)dx = xf'(x) - f(x) + C.$$

$$\text{So } \int_1^4 xf''(x)dx = xf'(x) - f(x)\Big|_1^4 = [4f'(4) - f(4)] - [1f'(1) - f(1)] = [4(1.1) - 2.6] - [0.7 - 2.4]$$

**Answer:** $\int_1^4 xf''(x)\,dx =$ 3.5

**b.** [3 points] Let $H(x) = \int_x^{x^2+1} f'(3t)\,dt$. Compute $H'(1)$.

$$H(x) = \frac{1}{3}f(3t)\Big|_x^{x^2+1} = \frac{1}{3}f(3x^2+3) - \frac{1}{3}f(3x)$$

$$\text{so } H'(x) = \frac{1}{3}f'(3x^2+3)(6x) - f'(3x)$$

$$\text{so } H'(1) = \frac{1}{3}f'(6)(6) - f'(3)$$

**Answer:** $H'(1) =$ 4.5

**c.** [3 points] Use TRAP(3) to approximate $\int_0^6 f(x)\,dx$. Write out each term in your sum.

$$\text{TRAP}(3) = \Delta x\left[\frac{1}{2}y_0 + y_1 + y_2 + \frac{1}{2}y_3\right] = (2)\left[\frac{1}{2}f(0) + f(2) + f(4) + \frac{1}{2}f(6)\right]$$

$$= 2\left[\frac{1}{2}(1) + 2.5 + 2.6 + \frac{1}{2}(6.7)\right]$$

**Answer:** $\int_0^6 f(x)\,dx \approx$ 17.9

**d.** [3 points] Find the 2nd degree Taylor polynomial $P_2(x)$ for $f(x)$ centered at $x = 3$.

$$P_2(x) = f(3) + f'(3)(x-3) + \frac{1}{2}f''(3)(x-3)^2$$

$$= 2.2 + (-0.1)(x-3) + \frac{1}{2}(0.8)(x-3)^2$$

**Answer:** $P_2(x) =$ $2.2 - 0.1(x-3) + 0.4(x-3)^2$

**e.** [3 points] Use your answer to part (d) to approximate $\int_0^6 f(x)\,dx$.

$$\int_0^6 f(x)dx \approx \int_0^6 P_2(x)\,dx = \int_0^6 (2.2 - 0.1(x-3) + 0.4(x-3)^2)dx$$

let $w = x-3$

$dw = dx$

$x = 0 \Rightarrow w = -3$

$x = 6 \Rightarrow w = 3$

$$= \int_{-3}^3 (2.2 - 0.1w + 0.4w^2)\,dw = 2.2w - 0.05w^2 + \frac{0.4}{3}w^3\Big|_{-3}^3$$

**Answer:** $\int_0^6 f(x)\,dx \approx$ 20.4