1. [15 points] The table below gives several values of a twice differentiable function $f$ along with its derivative $f'$ and continuous second derivative $f''$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>2.4</td>
<td>2.5</td>
<td>2.2</td>
<td>2.6</td>
<td>4.3</td>
<td>6.7</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>2</td>
<td>0.7</td>
<td>-0.3</td>
<td>-0.1</td>
<td>1.1</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>$f''(x)$</td>
<td>-1</td>
<td>-1.4</td>
<td>-0.5</td>
<td>0.8</td>
<td>1.4</td>
<td>0.7</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

Unless otherwise stated, you do not have to show work, but work shown might be considered for partial credit.

a. [3 points] Find the value of $\int_1^4 x f''(x) \, dx$.

\[
\int_1^4 x f''(x) \, dx = \int_1^4 (x \cdot f''(x)) \, dx = \int_1^4 (x \cdot 4) \, dx = \left[ 4x \right]_1^4 = 16 - 4 = 12
\]

Answer: $\int_1^4 x f''(x) \, dx = 12$

b. [3 points] Let $H(x) = \int_0^{x^2+1} f'(3t) \, dt$. Compute $H'(1)$.

\[
H(x) = \left. \frac{1}{3} f(3x^4+3) \right|_0^4 = \frac{1}{3} f(3(4)^4+3) - \frac{1}{3} f(3(0))
\]

So $H'(x) = \frac{1}{3} f'(3x^4+3)(6x) - f'(3x)$

So $H'(1) = \frac{1}{3} f'(6) - f'(3)$

Answer: $H'(1) = 4.5$

c. [3 points] Use TRAP(3) to approximate $\int_0^6 f(x) \, dx$. Write out each term in your sum.

\[
\text{TRAP}(3) = \Delta x \left[ \frac{1}{2} y_0 + y_1 + y_2 + \frac{1}{2} y_3 \right] = 2 \left[ \frac{1}{2} (2.2) + 2.5 + 2.6 + \frac{1}{2} (6.7) \right]
\]

Answer: $\int_0^6 f(x) \, dx \approx 17.9$

d. [3 points] Find the 2nd degree Taylor polynomial $P_2(x)$ for $f(x)$ centered at $x = 3$.

\[
P_2(x) = f(3) + f'(3)(x-3) + \frac{1}{2} f''(3)(x-3)^2
\]

$\approx 2.2 + (-0.1)(x-3) + \frac{1}{2} (0.8)(x-3)^2$

Answer: $P_2(x) = 2.2 - 0.1(x-3) + 0.4(x-3)^2$

e. [3 points] Use your answer to part (d) to approximate $\int_0^6 f(x) \, dx$.

\[
\int_0^6 f(x) \, dx \approx \int_0^6 P_2(x) \, dx = \int_0^6 \left[ 2.2 - 0.1(x-3) + 0.4(x-3)^2 \right] \, dx
\]

$\approx \int_0^3 \left[ 2.2 - 0.4(x-3)^2 \right] \, dx = \left[ 2.2x - 0.08(x-3)^3 \right]_0^3
\]

Answer: $\int_0^6 f(x) \, dx \approx 20.4$