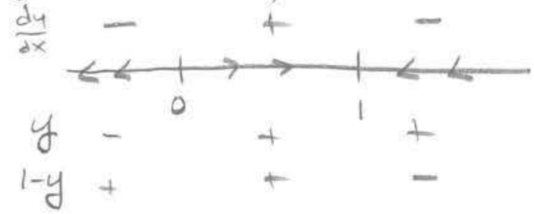
11. [9 points] Provide an example for each of the following. Your example must clearly satisfy the given properties. If no example exists then write DOES NOT EXIST and briefly explain why no such example exists.

a. [3 points] A differential equation that has at least one equilibrium solution that is stable and at least one equilibrium solution that is not stable. (A complete answer consists of the differential equation and both of these equilibrium solutions.)

$$\frac{dy}{dx} = y(1-y)$$

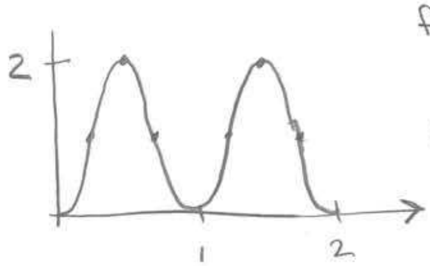


Answer: Differential Equation: dy/dx = $y(1-y)$

Equilibrium Solutions: Stable: 1 Not Stable: 0

b. [3 points] A continuous function $f(x)$ such that $\text{LEFT}(2) \le \text{RIGHT}(2) \le \int_0^2 f(x)\,dx$ where LEFT(2) and RIGHT(2) are, respectively, the left- and right- hand Riemann sum estimates for $\int_0^2 f(x)\,dx$ with two equal subintervals.

You may describe your function f by giving a formula or by drawing a clear and well-labeled graph. Then briefly explain why your function is indeed such an example.



$f(x) = 1 - \cos(2\pi x)$

$\text{LEFT}(2) = (1)\,[f(0) + f(1)] = 0$

$\text{RIGHT}(2) = (1)\,[f(1) + f(2)] = 0$

But $\int_0^2 f(x)\,dx = x - \frac{1}{2\pi}\sin(2\pi x)\Big|_0^2 = [2-0]-[0-0] = 2$

Answer: $f(x) =$ $1 - \cos(2\pi x)$

<u>**Brief**</u> **explanation:**

If f is neither increasing nor decreasing, it's possible for both LEFT and RIGHT to be overestimates or underestimates. In this case, the sample points happen to be the lowest values of the function.

c. [3 points] Give an example of a power series whose radius of convergence is 0. (You must use sigma notation for full credit.)

How about: $\sum_{n=0}^{\infty} n!\,x^n$. By the ratio test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!\,x^{n+1}}{n!\,x^n}\right|$

$= \lim_{n\to\infty} (n+1)|x|$ which diverges unless $x = 0$.

Answer: Power Series: $\sum_{n=0}^{\infty} n!\,x^n$