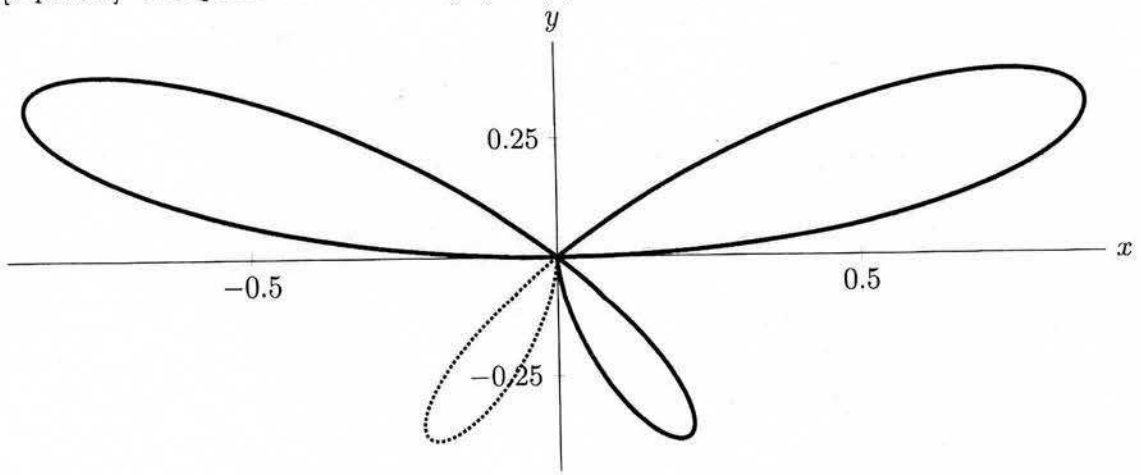
4. [9 points] The polar curve $r = \sin(4\theta)\cos(\theta)$ for $0 \le \theta \le \pi$ is shown below.



Note that there are two "large loops" and two "small loops".

For reference, note that for this curve, $\dfrac{dr}{d\theta} = 4\cos(\theta)\cos(4\theta) - \sin(\theta)\sin(4\theta)$

a. [3 points] For what values of θ does the polar curve $r = \sin(4\theta)\cos(\theta)$ trace once around the "small loop" in the third quadrant? (This portion of the curve is indicated by the dotted line.) Give your answer as an interval of θ values between 0 and π.

Look at signs of x and y to determine quadrant of points:

θ	sin θ	cos θ	sin 4θ	r = sin4θcosθ	x = rcosθ	y = rsinθ	
0	+	+	+	+	+	+	Q1
π/4	+	+	−	−	−	−	Q3
π/2	+	−	+	−	+	−	Q4
3π/4	+	−	−	+	−	+	Q2
π							

Answer: $\dfrac{\pi}{4} < \theta < \dfrac{\pi}{2}$

b. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the total arc length of the two small loops.

$$\text{Arc len} = 2\int_{\pi/4}^{\pi/2} \sqrt{r^2 + (r')^2}\, d\theta$$

Answer: Arc Length $= 2\displaystyle\int_{\pi/4}^{\pi/2} \sqrt{(\sin 4\theta\cos\theta)^2 + (4\cos 4\theta\cos\theta - \sin 4\theta\sin\theta)^2}\, d\theta$

c. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the area of the region that is enclosed by the polar curve $r = 2$ but is outside the curve $r = \sin(4\theta)\cos(\theta)$.

$|r| = |\sin 4\theta| \cdot |\cos\theta| \le 1 \cdot 1 < 2$, so the butterfly is contained in the circle of radius 2.

Area inside butterfly $= \frac{1}{2}\int r^2 d\theta = \frac{1}{2}\int_0^{\pi} \sin^2 4\theta \cos^2\theta\, d\theta$

Answer: Area $= 4\pi - \frac{1}{2}\displaystyle\int_0^{\pi} \sin^2 4\theta \cos^2\theta\, d\theta$