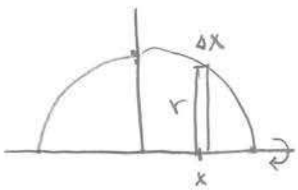
6. [6 points] Consider the curve $y = \sqrt{1-x^2}$. Suppose a paperweight is formed by rotating this curve around the x-axis. This paperweight has a density given by $\rho(x) = 2 + \cos(x)$ g/cm^3. The units on both axes are centimeters (cm).

a. [3 points] Write an expression that gives the approximate mass, in grams, of a slice of the paperweight taken perpendicular to the x-axis at coordinate x with thickness Δx. (Assume that Δx is small but positive.) Your expression should not involve any integrals.



radius of slice $= \sqrt{1-x^2}$ cm

Volume of slice $= \pi r^2 \Delta x = \pi(1-x^2)\Delta x$ cm^3

mass of slice $= \rho(x) \cdot \text{vol}$

Answer: Mass of slice $\approx$ $(2+\cos x)\,\pi(1-x^2)\,\Delta x$

b. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the mass, in grams, of the paperweight.

Answer: Mass $= \int_{-1}^{1} (2+\cos x)\,\pi(1-x^2)\,dx$

7. [6 points] Determine whether the following series converges absolutely, converges conditionally, or diverges, and give a complete argument justifying your answer. In particular, be sure to show all work and include any convergence tests used.

Correction made at time of exam

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$$

Circle one: CONVERGES ABSOLUTELY $\boxed{\text{CONVERGES CONDITIONALLY}}$ DIVERGES

Justification:

- terms alternate in sign
- |terms| decreases
- |terms| $\to 0$.

So converges by the alternating series test.

But $\frac{\ln(n)}{n} \geq \frac{1}{n}$ eventually, and $\sum \frac{1}{n}$ diverges by the p-test ($p=1$). So $\sum \frac{\ln(n)}{n}$ diverges by comparison