8. [8 points] Suppose Xena the cat is running around the backyard while wearing a tracking device on her collar. The device measures her $x$- and $y$-coordinates in meters, with the origin set as the center of the yard. Suppose $t$ is measured in minutes after Xena went outside. Xena's $x$- and $y$-coordinates satisfy the following differential equations for $t > 0$:

$$\frac{dx}{dt} = \cos(x) \sqrt{t^2 + \cos^2(x)}$$  \quad and \quad $$\frac{dy}{dt} = -\sin(y) \sqrt{t^2 + \sin^2(y)}$$

Portions of the slope fields for these differential equations are shown below.

Note the following: One minute after she goes outside, Xena is at the point $(\pi, 3\pi/2)$.

a. [3 points] How fast is Xena traveling one minute after she goes outside?

$$\left. \frac{dx}{dt} \right|_{t=1} = \cos(\pi) \sqrt{1^2 + \cos^2(\pi)} = (-1) \sqrt{1 + 1} = -\sqrt{2}$$

$$\left. \frac{dy}{dt} \right|_{t=1} = -\sin(3\pi/2) \sqrt{1 + \sin^2(3\pi/2)} = -1 \sqrt{1 + 1} = -\sqrt{2}$$

Answer: Speed = $\frac{2}{\sqrt{2}} = \sqrt{2}$ meters per minute

b. [3 points] Find the equation (in $xy$-coordinates) of the line tangent to Xena's path one minute after she goes outside.

$$\frac{dy}{dx} = \frac{dx/dt}{dy/dt}$$  so  $a + t = 1$  \quad $\frac{dy}{dx} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$

Answer: $y = (-1) (x - \pi) + \frac{3\pi}{2} = \frac{\pi}{2} - x$

c. [2 points] If Xena keeps running around the yard for a long time, what point (in $xy$-coordinates) in the yard will she approach?

Answer: $\left( \frac{\pi}{2}, 2\pi \right)$