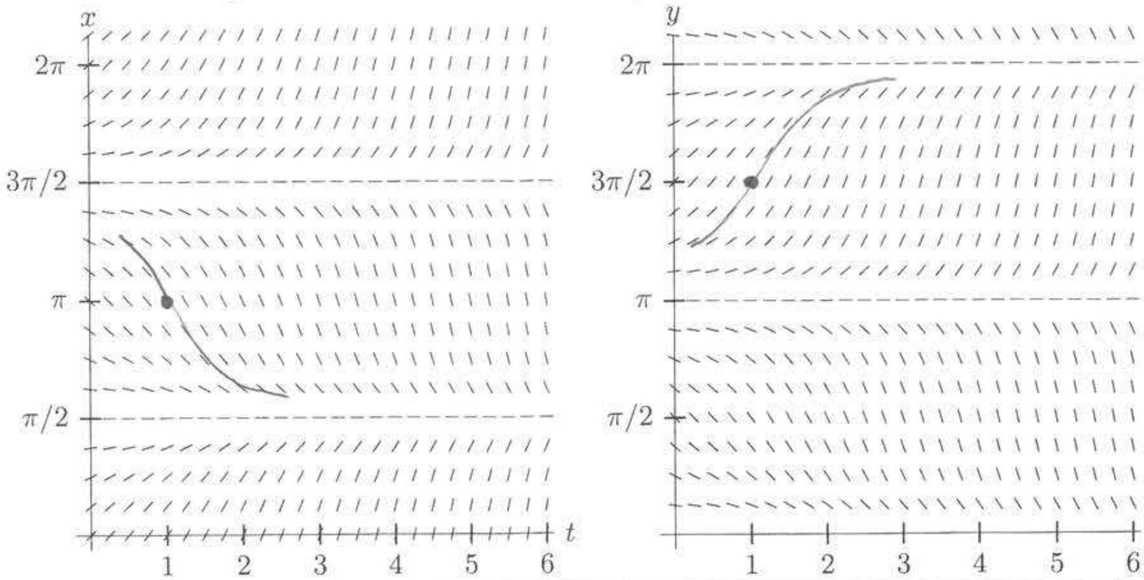
8. [8 points] Suppose Xena the cat is running around the backyard while wearing a tracking device on her collar. The device measures her x- and y-coordinates in meters, with the origin set as the center of the yard. Suppose t is measured in minutes after Xena went outside. Xena's x- and y- coordinates satisfy the following differential equations for $t > 0$:

$$\frac{dx}{dt} = \cos(x)\sqrt{t^2 + \cos^2(x)} \quad \text{and} \quad \frac{dy}{dt} = -\sin(y)\sqrt{t^2 + \sin^2(y)}$$

Portions of the slope fields for these differential equations are shown below.



Note the following: $\boxed{\text{One minute after she goes outside, Xena is at the point } (\pi, 3\pi/2).}$

a. [3 points] How fast is Xena traveling one minute after she goes outside? So $x = \pi$, $y = \frac{3\pi}{2}$

$\left.\frac{dx}{dt}\right|_{t=1} = \cos(\pi)\sqrt{1^2 + \cos^2\pi} = (-1)\sqrt{1^2+(-1)^2} = -\sqrt{2}$

$\left.\frac{dy}{dt}\right|_{t=1} = -\sin\left(\frac{3\pi}{2}\right)\sqrt{1^2 + \sin^2\left(\frac{3\pi}{2}\right)} = -(-1)\sqrt{1^2 + (-1)^2} = \sqrt{2}$

Speed $= \sqrt{x'^2 + y'^2} = \sqrt{2+2} = 2$

Answer: Speed = 2 meters per minute

b. [3 points] Find the equation (in xy-coordinates) of the line tangent to Xena's path one minute after she goes outside.

$\frac{dy}{dx} = \frac{dx/dt}{dy/dt}$, so at $t=1$ $\frac{dy}{dx} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$

Answer: $y = (-1)(x - \pi) + \frac{3\pi}{2} = \frac{5\pi}{2} - x$

c. [2 points] If Xena keeps running around the yard for a long time, what point (in xy-coordinates) in the yard will she approach?

Answer: $\left(\frac{\pi}{2}, 2\pi\right)$