6. [11 points]
   a. [7 points] Determine whether the following series converges or diverges. Be sure to fully justify your answer, showing all work and indicating any theorems you use.
   \[ \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^3 + 5} \]
   Answer (Circle one): \hspace{1cm} Diverges \hspace{1cm} Converges
   Justification:

   b. [4 points] Let \( f(x) \) be a positive, decreasing function on \([1, \infty)\) with \( \lim_{x \to \infty} f(x) = 1 \), and let \( a_n = f(n) \) and \( S_n = a_1 + \cdots + a_n \) for all \( n \geq 1 \). Decide whether the following converge, diverge, or if it cannot be determined. No justification is necessary.
   (i) The integral \( \int_{1}^{\infty} f(x) \, dx \)
       Diverges \hspace{1cm} Converges \hspace{1cm} CANNOT BE DETERMINED
   (ii) The sequence \( a_n \)
       Diverges \hspace{1cm} Converges \hspace{1cm} CANNOT BE DETERMINED
   (iii) The sequence \( S_n \)
       Diverges \hspace{1cm} Converges \hspace{1cm} CANNOT BE DETERMINED
   (iv) The series \( \sum_{n=1}^{\infty} \frac{1}{a_n} \)
       Diverges \hspace{1cm} Converges \hspace{1cm} CANNOT BE DETERMINED