

8. [9 points] Derivative Girl and Gradi-Ant are excited for the end of the semester. To celebrate, they decide to make an Infinite Party Horn. In this problem, x and y are measured in meters. (In Derivative Girl's world, infinite objects are possible.)
- a. [4 points] They decide to make the horn by rotating the region bounded by the positive x -axis, the positive y -axis, and the function $y = \frac{1}{2(x+1)^2}$ about the line $y = -1$. Write, but do not evaluate, an expression involving one integral that gives the volume, in cubic meters, of the Infinite Party Horn.

Answer: _____

- b. [5 points] Derivative Girl will use her favorite continuous and differentiable functions f and g to make a banner for the Infinite Party Horn. She loves the functions f and g because they have the properties:

$$\begin{array}{ll} \bullet \frac{d}{dx} \left(\frac{1+x}{g(x)} \right) = f(x), & \bullet \lim_{x \rightarrow \infty} g(x) = \infty, \\ \bullet g(1) = 15, & \bullet \lim_{x \rightarrow \infty} g'(x) = 5, \end{array}$$

and the area of the banner, in square meters, is given by

$$\int_1^{\infty} 20f(x) dx.$$

Does the banner have finite area? If so, what is the banner's area? Show all work and indicate any theorems you use.

Answer (Circle one): **Infinite area** **Finite area:** _____ m^2