8. [9 points] Derivative Girl and Gradi-Ant are excited for the end of the semester. To celebrate, they decide to make an Infinite Party Horn. In this problem, $x$ and $y$ are measured in meters. (In Derivative Girl’s world, infinite objects are possible.)

a. [4 points] They decide to make the horn by rotating the region bounded by the positive $x$-axis, the positive $y$-axis, and the function $y = \frac{1}{2(x+1)^2}$ about the line $y = -1$. Write, but do not evaluate, an expression involving one integral that gives the volume, in cubic meters, of the Infinite Party Horn.

Answer: 

b. [5 points] Derivative Girl will use her favorite continuous and differentiable functions $f$ and $g$ to make a banner for the Infinite Party Horn. She loves the functions $f$ and $g$ because they have the properties:

- $\frac{d}{dx} \left( \frac{1 + x}{g(x)} \right) = f(x)$,
- $g(1) = 15$,
- $\lim_{x \to \infty} g(x) = \infty$,
- $\lim_{x \to \infty} g'(x) = 5$,

and the area of the banner, in square meters, is given by

$$\int_{1}^{\infty} 20f(x) \, dx.$$ 

Does the banner have finite area? If so, what is the banner’s area? Show all work and indicate any theorems you use.

Answer (Circle one): Infinite area   Finite area: ____________ m²