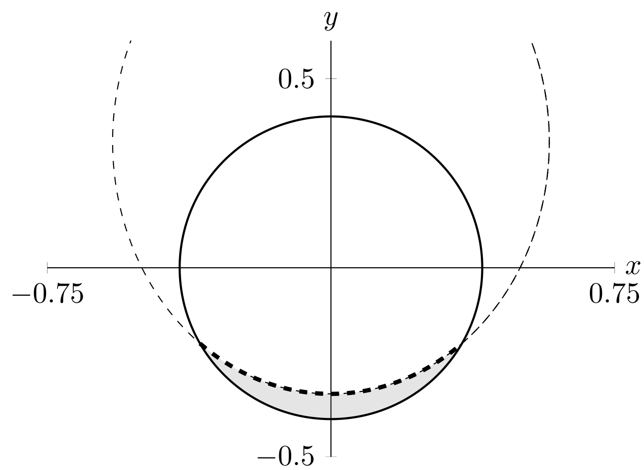
2. [9 points] Scientists are studying the bite of several different rodents. To do this, they give a wafer cookie to the animal, and take it away after the animal takes one bite.

- r is measured in inches
- The **wafer** is modeled by the region inside the polar curve
$$r = \frac{2}{5}$$
(the **solid** line in the diagram).
- The rodent's **bite** is modeled by the region inside the polar curve
$$r = \frac{1}{2 - \sin(\theta)}$$
and inside the wafer (the **dashed** line in the diagram).
- The wafer remaining after the bite is shaded in the diagram.



a. [3 points] For what values of θ between 0 and 2π does the rodent's bite meet the edge of the wafer? Justify your answer algebraically, and give your answers in **exact** form.

Solution: The bite meets the edge of the wafer when
$$\frac{2}{5} = \frac{1}{2 - \sin(\theta)},$$
which happens when $\sin(\theta) = \frac{-1}{2}$, giving us $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$.
Note that $\arcsin(-1/2) < 0$, so this does not satisfy the requirements of the problem. However, we could use this to find that $\pi - \arcsin(-1/2)$ and $2\pi + \arcsin(-1/2)$ are both between 0 and 2π.

Answer: $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

b. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the area, in square inches, of the wafer remaining after the bite.

Answer: $\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{4}{25} - \frac{1}{(2-\sin(\theta))^2} d\theta$

c. [3 points] The bite mark in the wafer is represented by the **thick** dashed line in the diagram. Write, but do not evaluate, an expression involving one or more integrals that gives the length, in inches, of this bite mark.

Solution: We use the formula for arc length and
$$\frac{dr}{d\theta} = \frac{\cos(\theta)}{(2-\sin(\theta))^2}.$$

Answer: $\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \sqrt{\frac{1}{(2-\sin(\theta))^2} + \frac{\cos^2(\theta)}{(2-\sin(\theta))^4}} d\theta$