5. [13 points] Two particles move in the xy-plane. At time $t > 0$, the position of particle A is given by

$$\begin{cases} x(t) = -6 - 3t \\ y(t) = 2t - k \end{cases}$$

and the position of particle B is

$$\begin{cases} x(t) = -4t \\ y(t) = t^2 - t - 8 \end{cases}$$

For the following questions, justify your answers algebraically.

a. [4 points] Find k so that the two particles must collide.

Solution: The two particles have the same x coordinate when $-6-3t = -4t$, so when $t = 6$. To ensure they collide, at $t = 6$, they must have the same y coordinate, so:

$$2(6) - k = 6^2 - 6 - 8,$$

giving us $k = -10$.

Answer: $k =$ -10

b. [3 points] At the time the particles collide, which is moving faster?

Solution: We have $x'(t) = -3$ and $y'(t) = 2$ for particle A; $x'(t) = -4$ and $y'(t) = 2t - 1$ for particle B. So, the speed at $t = 6$ for particle A is $\sqrt{9+4} = \sqrt{13}$, and for particle B is $\sqrt{16 + (2(6) - 1)^2} = \sqrt{137}$.

Answer: PARTICLE A PARTICLE B

c. [3 points] Use MID(2) to approximate the length of the path traveled by particle B between $t = 0$ and $t = 4$. Write out all the terms in your sum.

Solution: Our integral is $\int_0^4 \sqrt{16 + (2t-1)^2}dt$.

Answer: $2\left(\sqrt{16 + (2(1) - 1)^2}\right) + 2\left(\sqrt{16 + (2(3) - 1)^2}\right)$

d. [3 points] For what positive values of t is the slope of the tangent line to the path of particle B positive?

Solution: The slope of the tangent line at t is $\frac{2t-1}{-4}$, which is positive when $2t - 1$ is negative, which is when $t < \frac{1}{2}$.

Answer: $0 < t < \frac{1}{2}$

