6. [11 points]

a. [7 points] Determine whether the following series converges or diverges. Be sure to fully justify your answer, showing all work and indicating any theorems you use.

$$\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^3+5}$$

Answer *(Circle one):* **Diverges** **Converges**

Justification:

Solution: We have $\frac{|\sin(2n)|}{n^3+5} \leq \frac{1}{n^3}$. Since $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by $p-$test (as $p = 3 > 1$), $\sum_{n=1}^{\infty} \frac{|\sin(2n)|}{n^3+5}$ converges by comparison test. So $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^3+5}$ converges absolutely.

b. [4 points] Let $f(x)$ be a positive, decreasing function on $[1, \infty)$ with $\lim_{x\to\infty} f(x) = 1$, and let $a_n = f(n)$ and $S_n = a_1 + \cdots + a_n$ for all $n \geq 1$.
Decide whether the following converge, diverge, or if it cannot be determined. No justification is necessary.

(i) The integral $\int_1^{\infty} f(x)\,dx$

[Diverges] **Converges** CANNOT BE DETERMINED

(ii) The sequence a_n

Diverges **[Converges]** CANNOT BE DETERMINED

(iii) The sequence S_n

[Diverges] **Converges** CANNOT BE DETERMINED

(iv) The series $\sum_{n=1}^{\infty} \frac{1}{a_n}$

[Diverges] **Converges** CANNOT BE DETERMINED