

8. [9 points] Derivative Girl and Gradi-Ant are excited for the end of the semester. To celebrate, they decide to make an Infinite Party Horn. In this problem,  $x$  and  $y$  are measured in meters. (In Derivative Girl's world, infinite objects are possible.)

- a. [4 points] They decide to make the horn by rotating the region bounded by the positive  $x$ -axis, the positive  $y$ -axis, and the function  $y = \frac{1}{2(x+1)^2}$  about the line  $y = -1$ . Write, but do not evaluate, an expression involving one integral that gives the volume, in cubic meters, of the Infinite Party Horn.

**Answer:**  $\pi \int_0^{\infty} \left( \frac{1}{2(x+1)^2} + 1 \right)^2 - 1 \, dx$

- b. [5 points] Derivative Girl will use her favorite continuous and differentiable functions  $f$  and  $g$  to make a banner for the Infinite Party Horn. She loves the functions  $f$  and  $g$  because they have the properties:

- $\frac{d}{dx} \left( \frac{1+x}{g(x)} \right) = f(x),$
- $\lim_{x \rightarrow \infty} g(x) = \infty,$
- $g(1) = 15,$
- $\lim_{x \rightarrow \infty} g'(x) = 5,$

and the area of the banner, in square meters, is given by

$$\int_1^{\infty} 20f(x) \, dx.$$

Does the banner have finite area? If so, what is the banner's area? Show all work and indicate any theorems you use.

*Solution:* Using the information we've been given, we find

$$\begin{aligned} \int_1^{\infty} 20f(x) \, dx &= \lim_{b \rightarrow \infty} \int_1^b 20f(x) \, dx \\ &= \lim_{b \rightarrow \infty} \frac{20 + 20b}{g(b)} - \frac{40}{g(1)} \\ &= \lim_{b \rightarrow \infty} \frac{20}{g'(b)} - \frac{40}{15} \text{ by L'Hopital's Rule} \\ &= \frac{20}{5} - \frac{40}{15} \end{aligned}$$

**Answer (Circle one):**    **Infinite area**    **Finite area:**  $\frac{4}{3}$  m<sup>2</sup>