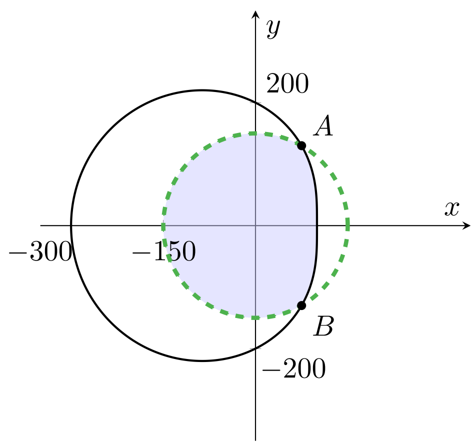
2. [15 points] With a crashing stork market, the infinite trumpet glitch, and the forestry expansion over-expanding, the video game *Vegetable Crossing* has a lot of issues. Maria designs a new island for the game, and on the island there is an area where players can grow acacia plants.

- The island is in the shape of the polar curve $r = 200 - 100\cos(\theta)$ where $0 \le \theta < 2\pi$. The outline of the island is the **solid black curve** plotted below.
- **The acacia-growing zone is shaded blue**, and it is formed by the section of the island inside a circle of radius 150 meters centered at the origin. The circle is the dashed green curve plotted below.
- All distances on the graph are in meters.



a. [5 points] The points A and B, labeled above, are the intersection points of the polar curve $r = 200 - 100\cos(\theta)$ with the dashed green circle. Find points A and B **in polar coordinates**.

Solution: For the points of intersection, $200 - 100\cos(\theta) = 150$, and so $\cos(\theta) = \frac{1}{2}$. This means we must have $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$. Therefore, $A = (150, \frac{\pi}{3})$ and $B = (150, \frac{5\pi}{3})$.

b. [5 points] Find an expression involving one or more integrals for the length, in meters, of the perimeter of the acacia-growing zone. Do not evaluate your integral(s).

Solution: Part of the perimeter, is an arc of the circle of radius 150. The arc length of the section which is within the island is two-thirds $\left(\left(\frac{5\pi}{3} - \frac{\pi}{3}\right)/(2\pi)\right)$ of the circumference of the circle, and the circumference of the entire circle is 300π, so the arc length of this section is 200π.
For $f(\theta) = 200 - 100\cos(\theta)$, we have $f'(\theta) = 100\sin(\theta)$, and so the arc length of this section of the polar curve is given by

$$2\int_0^{\pi/3} \sqrt{(200 - 100\cos(\theta))^2 + (100\sin(\theta))^2}\, d\theta$$

where we have used the symmetry of the region.
In total then, the length of the perimeter is

$$200\pi + 2\int_0^{\pi/3} \sqrt{(200 - 100\cos(\theta))^2 + (100\sin(\theta))^2}\, d\theta.$$

c. [5 points] Players are able to pave any part of the island **outside** of the acacia-growing zone, at a cost of 7 dubloons per square meter. Find an expression involving one or more integrals for the cost, in dubloons, of paving the entire area which lies outside of the acacia-growing zone. Do not evaluate your integral(s).

Solution: The area outside the acacia-growing zone can be thought of as the area of the sector of the island with $\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}$ with the area inside the circle in this sector subtracted. The area of this sector of the circle is again two-thirds of the area of the circle, which is equal to $\frac{2}{3}\pi(150)^2$, and so the total area of the acacia-growing zone is

$$\frac{1}{2}\int_{\pi/3}^{5\pi/3} (200 - 100\cos(\theta))^2\ d\theta - \frac{2}{3}\pi(150)^2.$$

This means that the cost, in dubloons, is

$$7\left(\frac{1}{2}\int_{\pi/3}^{5\pi/3} (200 - 100\cos(\theta))^2\ d\theta - \frac{2}{3}\pi(150)^2\right).$$