

4. [16 points] Buluu and Nyekundu the giraffes work on another art project. They paint on a large, square canvas, with corners at $(0, 0)$, $(0, 100)$, $(100, 0)$, and $(100, 100)$, with distances in yards. As Buluu moves, he leaves a trail of blue paint. As Nyekundu moves, she leaves a trail of red paint.

Buluu's position t minutes after the giraffes start painting is described by the parametric equations:

$$\begin{cases} x(t) = t^2 \\ y(t) = 40 + 2(t - 5)^2 \end{cases}$$

and Nyekundu's position t minutes after the giraffes start painting is described by the parametric equations:

$$\begin{cases} x(t) = 25 + 20 \sin(\pi t) \\ y(t) = 40 - (t - 5)^2 \end{cases}$$

- a. [5 points] If the giraffes ever collide, they produce a purple splotch on the canvas at the point of collision. Will the giraffes produce any purple splotches? If so, at **what time(s)** does this occur, and **where on the canvas** are the purple splotches located?

Solution: Notice that for Buluu, $y(t) \geq 40$, and for Nyekundu, $y(t) \leq 40$, so their y -coordinates are only equal when $y(t) = 40$. This happens for both giraffes when $t = 5$. At this time, both have $x(t) = 25$, and so there is a purple splotch created at $t = 5$ at $(25, 40)$.

- b. [6 points] Find an expression for Buluu's speed 3 minutes after the giraffes start painting. Make sure to include units.

Solution: For Buluu, $\frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 4(t - 5)$, and so Buluu's speed at $t = 3$ is $\sqrt{(2(3))^2 + (4(-2))^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ yards per minute

- c. [5 points] At what time does Nyekundu first leave the canvas? Make sure to fully justify your answer.

Solution: For Nyekundu, $5 \leq x(t) \leq 45$, and so her x -coordinate is never less than 0 or bigger than 100.

She also has $y(t) \leq 40$, and so she leaves the canvas when $y(t) = 0$, i.e when $(t - 5)^2 = 40$. Solving gives $t = 5 + \sqrt{40}$ as the only (positive) time when this happens. Since $y(t)$ is negative for all larger values of t , Nyekundu does leave the canvas at this time.