

8. [6 points] Suppose

$$F(x) = \int_3^{2x} (e^{5t^2} - 2) dt.$$

Find all x -values where the graph of $y = F(x)$ has a horizontal tangent line, showing all of your work. You do not need to simplify your answer(s).

Solution: The graph has a horizontal tangent line where $F'(x) = 0$.

Using the Second Fundamental Theorem, we see $F'(x) = 2(e^{5(2x)^2} - 2) = 2(e^{20x^2} - 2)$, and so $F'(x) = 0$ when $e^{20x^2} = 2$, i.e. $20x^2 = \ln 2$.

Therefore, $x = \pm\sqrt{\frac{\ln 2}{20}}$.

9. [9 points] For each part of this problem, write the CAPITAL LETTER corresponding to **ALL** answers that apply on your submission. You do not need to show your work.

- a. [5 points] Suppose $f(x)$ is a continuous function defined for $x \geq 1$ satisfying:

- $f(x) > 0$ for all $x \geq 1$.
- $f(x)$ is decreasing on its domain.
- $f(x) \leq \frac{1}{\sqrt{x}}$

Which of the following **MUST** be true about $f(x)$?

(A) $\int_1^{\infty} f(x) dx$ converges.

(B) $\int_1^{\infty} (f(x))^2 dx$ converges.

(C) $\int_1^{\infty} \frac{f(x)}{x} dx$ converges.

(D) $\int_1^{\infty} f(x) dx$ diverges.

(E) None of the above.

- b. [4 points] Which of the following pairs of polar coordinates are the same point in the xy -plane as the point $(x, y) = (-1, 1)$?

(A) $(r, \theta) = \left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

(B) $(r, \theta) = \left(1, \frac{3\pi}{4}\right)$

(C) $(r, \theta) = \left(-\sqrt{2}, -\frac{\pi}{4}\right)$

(D) $(r, \theta) = \left(-\frac{\sqrt{2}}{2}, \frac{5\pi}{4}\right)$

(E) None of the above.