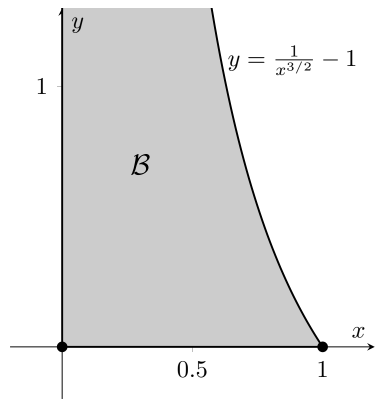
1. [9 points] Arnold is building a set for his son Michael's school play in which Michael will have to climb a very tall beanstalk to fight a giant.

a. [4 points] At first, Arnold decides that since the beanstalk is extremely tall, he should model it as an infinitely tall solid of revolution of the region $\mathcal{B}$ around the y-axis. Here, $\mathcal{B}$ is the unbounded region in the first quadrant to the left of the function $f(x) = \frac{1}{x^{3/2}} - 1$ for $0 < x \leq 1$, depicted partially below.



Write an integral for just the **area** of the region $\mathcal{B}$ (and not the rotated solid) in the space below. Determine whether your integral converges or diverges, with FULL JUSTIFICATION, and circle the word CONVERGES or DIVERGES corresponding to your conclusion.

Solution: The integral is

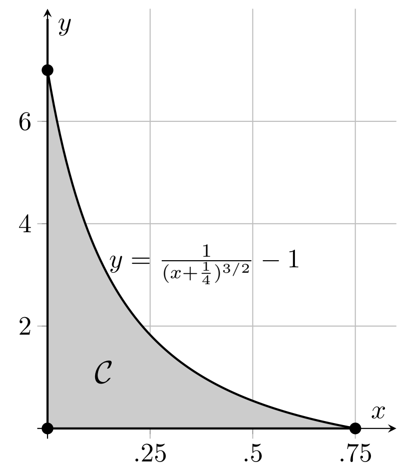
$$\int_0^1 \frac{1}{x^{3/2}} - 1dx = \int_0^1 \frac{1}{x^{3/2}}dx - 1.$$

Using the p-test ($p = \frac{3}{2}$), the integral $\int_0^1 \frac{1}{x^{3/2}}dx$ diverges, so the whole integral diverges.

The integral is $\int_0^1 \frac{1}{x^{3/2}} - 1dx$ and it CONVERGES / $\boxed{\text{DIVERGES}}$.

1. (continued)

b. [5 points] Arnold realizes modelling a beanstalk as infinitely tall is not the most realistic, so he changes his region to be $\mathcal{C}$. Here, the region $\mathcal{C}$ is bounded by the function $g(x) = \frac{1}{(x+\frac{1}{4})^{3/2}} - 1$, the x-axis, and the y-axis, depicted below.



If the model of the Beanstalk is now the solid formed by rotating the the region $\mathcal{C}$ around the y-axis, write, but do not solve, an integral that gives the **volume** of the beanstalk using the blank provided.

Solution: Taking vertical slices, we see that we obtain the shell method. The volume of a slice of thickness Δx at a horizontal coordinate x is approximately

$$\Delta V = 2\pi x \left(\frac{1}{(x+\frac{1}{4})^{3/2}} - 1 \right) \Delta x,$$

and so the total volume of the solid is

$$\int_0^{.75} 2\pi x \left(\frac{1}{(x+\frac{1}{4})^{3/2}} - 1 \right) dx.$$

Alternate Solution: Taking horizontal slices, we see that we obtain the disc method. The volume of a slice of thickness Δy at vertical coordinate y is approximately

$$\Delta V = \pi x^2 \Delta y = \pi \left((y+1)^{-2/3} - \frac{1}{4} \right)^2 \Delta y$$

and so now the total volume of the solid is

$$\int_0^{7} \pi \left((y+1)^{-2/3} - \frac{1}{4} \right)^2 dy.$$

The integral is $\underline{\int_0^{.75} 2\pi x \left(\frac{1}{(x+\frac{1}{4})^{3/2}} - 1 \right) dx}$.