2. [5 points] Suppose that the power series $\sum_{n=1}^{\infty} C_n(x+2)^n$ converges at $x = 4$ and diverges at $x = -10$. What can you say about the behavior of the power series at the following values of x?

a. [1 point] At $x = 0$, the power series...

$\boxed{\text{CONVERGES}}$ DIVERGES CANNOT DETERMINE

b. [1 point] At $x = -8$, the power series...

CONVERGES DIVERGES $\boxed{\text{CANNOT DETERMINE}}$

c. [1 point] At $x = 8$, the power series...

CONVERGES $\boxed{\text{DIVERGES}}$ CANNOT DETERMINE

d. [1 point] At $x = -4$, the power series...

$\boxed{\text{CONVERGES}}$ DIVERGES CANNOT DETERMINE

e. [1 point] At $x = 6$, the power series...

CONVERGES DIVERGES $\boxed{\text{CANNOT DETERMINE}}$

3. [7 points] A function $F(x)$ has Taylor series given by

$$F(x) = \sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{2^n(n^2+1)}(x-1)^{4n+1}$$

Answer the following questions regarding the Taylor series:

a. [3 points] Circle the appropriate answer. No work is needed, but partial credit may be given for correct work.

At $x = 1$, F is... $\boxed{\text{INCREASING}}$ DECREASING CANNOT DETERMINE

Solution: Using the Taylor series,

$$F'(1) = \frac{(-1)^0(1)}{2^0(1)} = 1 > 0,$$

so F is increasing at $x = 1$.

b. [4 points] What is $F^{(2021)}(1)$? Give your answer in exact form and do not try to simplify. Show your work.

Solution: Using the Taylor series, the term $(x-1)^{2021}$ appears when $n = 505$, so

$$\frac{F^{(2021)}(1)}{2021!} = \frac{(-1)^{505}(505+1)}{2^{505}(505^2+1)}$$

$F^{(2021)}(1) =$ $-\frac{(506)(2021)!}{2^{505}(505^2+1)}$.