**2.** [5 points] Suppose that the power series $\sum_{n=1}^{\infty} C_n(x+2)^n$ converges at $x=4$ and diverges at $x=-10$. What can you say about the behavior of the power series at the following values of $x$?

**a.** [1 point] At $x=0$, the power series...

[CONVERGES] DIVERGES CANNOT DETERMINE

**b.** [1 point] At $x=-8$, the power series...

CONVERGES DIVERGES [CANNOT DETERMINE]

**c.** [1 point] At $x=8$, the power series...

CONVERGES [DIVERGES] CANNOT DETERMINE

**d.** [1 point] At $x=-4$, the power series...

[CONVERGES] DIVERGES CANNOT DETERMINE

**e.** [1 point] At $x=6$, the power series...

CONVERGES DIVERGES [CANNOT DETERMINE]

**3.** [7 points] A function $F(x)$ has Taylor series given by

$$F(x)=\sum_{n=0}^{\infty}\frac{(-1)^n(n+1)}{2^n(n^2+1)}(x-1)^{4n+1}$$

Answer the following questions regarding the Taylor series:

**a.** [3 points] Circle the appropriate answer. No work is needed, but partial credit may be given for correct work.

At $x=1$, $F$ is... [INCREASING] DECREASING CANNOT DETERMINE

*Solution:* Using the Taylor series,

$$F'(1)=\frac{(-1)^0(1)}{2^0(1)}=1>0,$$

so $F$ is increasing at $x=1$.

**b.** [4 points] What is $F^{(2021)}(1)$? Give your answer in exact form and do not try to simplify. Show your work.

*Solution:* Using the Taylor series, the term $(x-1)^{2021}$ appears when $n=505$, so

$$\frac{F^{(2021)}(1)}{2021!}=\frac{(-1)^{505}(505+1)}{2^{505}(505^2+1)}$$

$F^{(2021)}(1)=$ $-\frac{(506)(2021)!}{2^{505}(505^2+1)}$.