4. [6 points] Find the **radius of convergence** of the following power series:

\[ \sum_{n=0}^{\infty} \frac{8^n(n!)^3}{(3n)!} (x - 5)^{3n}. \]

Show your work including full justifications of any tests you use.

**Solution:** Setting \( a_n = \frac{8^n(n!)^3}{(3n)!} (x - 5)^{3n} \), compute

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{8^{n+1} ((n + 1)!)^3}{(3n + 3)!} \frac{(3n)!}{8^n(n!)^3} |x - 5|^3
\]

\[
= \lim_{n \to \infty} \frac{(n + 1)^3}{8} \frac{1}{(3n + 3)(3n + 2)(3n + 1)} |x - 5|^3
\]

\[
= \frac{8}{27} |x - 5|^3.
\]

By the ratio test, the power series converges for

\[ \frac{8}{27} |x - 5|^3 < 1 \iff |x - 5| < \left( \frac{27}{8} \right)^{1/3} = \frac{3}{2}. \]

The radius of convergence is \( \frac{3}{2} \).