6. [13 points] Values of a function $g(x)$ and some of its derivatives at $x = 2$ are given in the table below. Use this information for some of the problems below.

$g(2)$	$g'(2)$	$g''(2)$	$g'''(2)$	$g^{(4)}(2)$
1	2	-4	0	4

a. [4 points] Find the first 4 nonzero terms of the Taylor series of $g(x)$ about $x = 2$. Write your final answer as a polynomial $P(x)$ in the blank below.

$P(x) =$ $1 + 2(x-2) - 2(x-2)^2 + \frac{1}{6}(x-2)^4$

b. [4 points] Using known Taylor series, find the first 3 nonzero terms of the Taylor series of $f(x) = (x-2)\ln\left(\frac{x}{2}\right)$ about $x = 2$. Write your final answer as a polynomial $Q(x)$ in the blank below. (*Hint:* $f(x) = (x-2)\ln\left(1 + \frac{(x-2)}{2}\right)$)

Solution: For $-1 < x \leq 1$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots,$$

about $x = 0$. So,

$$(x-2)\ln\left(1 + \frac{(x-2)}{2}\right) = (x-2)\left(\frac{x-2}{2} - \frac{1}{2}\left(\frac{x-2}{2}\right)^2 + \frac{1}{3}\left(\frac{x-2}{2}\right)^3 + \ldots\right)$$

$Q(x) =$ $\frac{(x-2)^2}{2} - \frac{(x-2)^3}{8} + \frac{(x-2)^4}{24}$

c. [5 points] Let $H(x) = 1 + \int_2^x f(t) + g(t)dt$. Find the first 4 nonzero terms of the Taylor series of H about $x = 2$. Write your final answer as a polynomial $R(x)$ in the blank below. Partial credit may be given for finding the appropriate terms of $\int_2^x f(t)dt$ or $\int_2^x g(t)dt$.

Solution: Putting together (a) and (b), R is the first 4 nonzero terms of $1 + \int_2^x P(t) + Q(t)dt$. Note that we only need the first 3 terms of P and the first term of Q:

$$\begin{aligned} 1 + \int_2^x P(t) + Q(t)dt &= 1 + \left[t + (t-2)^2 - \frac{2}{3}(t-2)^3\right]\Big|_2^x + \left[\frac{(t-2)^3}{6}\right]\Big|_2^x \\ &= 1 + (x-2) + (x-2)^2 - \frac{2}{3}(x-2)^3 + \frac{(x-2)^3}{6}. \end{aligned}$$

$R(x) =$ $1 + (x-2) + (x-2)^2 - \frac{(x-2)^3}{2}$