2. [15 points] The parts of this problem are unrelated to each other. Be sure to show work for all parts, and circle your final answer.

a. [5 points] A leaking bag of sugar is lifted vertically from the ground to a height of 10 feet above the ground. The weight of the bag of sugar is \(6 - \sqrt{x}\) lbs when it has been lifted \(x\) feet above the ground. Find the work done lifting the bag, including units. Fully evaluate any integrals, but you do not need to simplify your answer.

Solution: The work is obtained by integrating the force over the distance the bag is lifted. The force on the bag is equal to its weight, so we have:

\[
\int_0^{10} (6 - \sqrt{x}) \, dx = 60 - \frac{2}{3} x^{3/2} \bigg|_0^{10}
\]

\[
= 60 - \frac{2}{3} 10^{3/2}.
\]

Answer: \(60 - \frac{2}{3} 10^{3/2}\) lbs·ft

b. [5 points] Write an expression involving one or more integrals that gives the volume of the solid obtained by rotating the region in the \(xy\)-plane bounded between the \(x\)-axis, the parabola \(y = x^2 + 1\), the line \(x = -1\) and the line \(x = 1\), about the line \(x = -2\). Do not evaluate your integral(s).

Solution: Using the shell method, the volume is

\[
\int_{-1}^{1} 2\pi (x + 2)(x^2 + 1) \, dx.
\]

Answer: \(\int_{-1}^{1} 2\pi (x + 2)(x^2 + 1) \, dx\)
c. [5 points] The function \( f(x) = x^4 + 5 \) can be rewritten in the form \( f(x) = (x + 1)^4 + A(x + 1)^3 + B(x + 1)^2 + C(x + 1) + D \), where \( A, B, C, D \) are constants. Find the values of \( A, B, C, D \) using Taylor series. Other methods used to find the constants will not be given credit.

\[
\begin{align*}
A &= -4 \\
B &= 6 \\
C &= -4 \\
D &= 6
\end{align*}
\]