2. [15 points] The parts of this problem are unrelated to each other. Be sure to show work for all parts, and circle your final answer.

a. [5 points] A leaking bag of sugar is lifted vertically from the ground to a height of 10 feet above the ground. The **weight** of the bag of sugar is $6-\sqrt{x}$ lbs when it has been lifted x feet above the ground. Find the work done lifting the bag, including units. Fully evaluate any integrals, but you do not need to simplify your answer.

Solution: The work is obtained by integrating the force over the distance the bag is lifted. The force on the bag is equal to its weight, so we have:

$$\begin{aligned}\int_0^{10}(6-\sqrt{x})\,dx &= 60-\frac{2}{3}x^{3/2}\Big|_0^{10}\\ &= 60-\frac{2}{3}10^{3/2}.\end{aligned}$$

Answer: $60-\frac{2}{3}10^{3/2}$ **lbs·ft**

b. [5 points] Write an expression involving one or more integrals that gives the volume of the solid obtained by rotating the region in the xy-plane bounded between the x-axis, the parabola $y=x^2+1$, the line $x=-1$ and the line $x=1$, about the line $x=-2$. Do not evaluate your integral(s).

Solution: Using the shell method, the volume is

$$\int_{-1}^{1} 2\pi(x+2)(x^2+1)\,dx.$$

Answer: $\int_{-1}^{1} 2\pi(x+2)(x^2+1)\,dx$

c. [5 points] The function $f(x) = x^4 + 5$ can be rewritten in the form $f(x) = (x+1)^4 + A(x+1)^3 + B(x+1)^2 + C(x+1) + D$, where A, B, C, D are constants. Find the values of A, B, C, D using Taylor series. Other methods used to find the constants will not be given credit.

$A=$ $\underline{\quad -4 \quad}$

$B=$ $\underline{\quad 6 \quad}$

$C=$ $\underline{\quad -4 \quad}$

$D=$ $\underline{\quad 6 \quad}$