**4.** [12 points] Katya and Miles are sailing in the ocean, which is represented by the $xy$-plane. Katya's position, $t$ hours after 12:00pm, is given by

$$x = 3t, \quad y = \sin\left(\frac{\pi t}{2}\right),$$

while Miles' position, $t$ hours after 12:00pm, is given by

$$x = t^2 + 2, \quad y = \cos(\pi t) - 1.$$

In this problem, $x$ and $y$ have units in kilometers. All above equations are valid for $0 \leq t \leq 6$.

**a**. [2 points] What is **Miles'** position at 3:00pm?

*Solution:* To find this, we plug $t = 3$ into the equations given for the $x$ and $y$ coordinates of Miles.

$x =$ **11** $\quad y =$ **−2**

**b**. [4 points] Will Katya and Miles ever collide during their journey? If so, at what time(s) will this occur? Justify your answer.

*Solution:* We first consider when their $x$-coordinates will be equal:

$$3t = t^2 + 2$$
$$t^2 - 3t + 2 = 0$$
$$(t-2)(t-1) = 0$$

has solutions at $t = 1, 2$. We test their $y$-coordinates at each of these times. At $t = 1$, Katya's $y$-coordinate is 1, and Miles' is $-2$, so they do not collide then. At $t = 2$, Katya's $y$-coordinate is 0, as is Miles'. Therefore they collide then.

The time(s) is/are **2**

**c**. [3 points] What is the slope of the tangent line to **Katya's** path at $t = 4$?

*Solution:* We have $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right)}{3}$, which at $t = 4$ evaluates to $\dfrac{\frac{\pi}{2}(1)}{3} = \dfrac{\pi}{6}$.

The slope is $\dfrac{\pi}{6}$

**d**. [3 points] Write an expression involving one or more integrals that gives the distance, in kilometers, **Miles** traveled between 1:00pm and 4:00pm. Do not evaluate your integral(s).

*Solution:* For this we use the parametric arc length formula $\displaystyle\int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ with

$$\frac{dx}{dt} = 2t$$
$$\frac{dy}{dt} = -\pi \sin(\pi t).$$

The distance is $\displaystyle\int_1^4 \sqrt{(2t)^2 + (\pi \sin(\pi t))^2}\, dt$