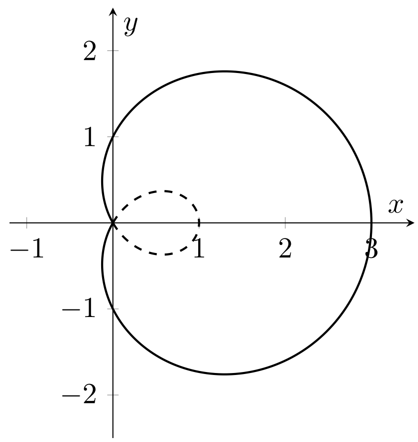
5. [11 points] The parts of this question relate to the following polar graph, defined by the polar curve $r(\theta) = -1 + 2\cos(\theta)$, on the domain $[0, 2\pi]$. Both the solid and dashed curves are part of the graph of $r(\theta)$.



a. [2 points] What are all the angles θ, with $0 \le \theta \le 2\pi$, for which the graph passes through the origin?

Solution: The curve passes through the origin when $r(\theta) = 0$, so we need to solve $\cos(\theta) = 1/2$. This occurs for θ in $[0, 2\pi]$ at the values $\theta = \pi/3, 5\pi/3$.

Answer(s): $\theta = \pi/3, 5\pi/3$

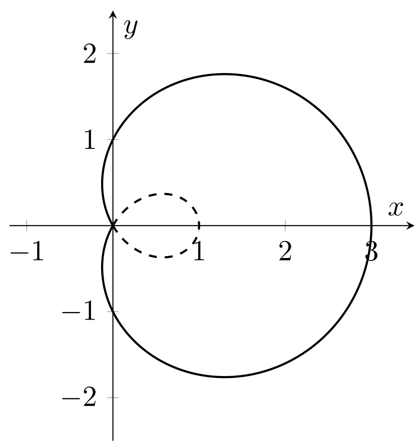
b. [2 points] Determine the interval(s) within $[0, 2\pi]$ for which θ traces out the **dashed** portion of the graph.

Answer(s): $[0, \pi/3]$ **and** $[5\pi/3, 2\pi]$

c. [3 points] Write, but do not evaluate, an expression involving one or more integrals which gives the **area** enclosed by the **dashed** portion of the graph.

The area is $\frac{1}{2}\int_0^{\pi/3}(-1+2\cos(\theta))^2\,d\theta + \frac{1}{2}\int_{5\pi/3}^{2\pi}(-1+2\cos(\theta))^2\,d\theta$

5. (continued) For your convenience, the polar graph referenced by this problem is reproduced here:



d. [4 points] Write, but do not evaluate, an expression involving one or more integrals which gives the **arc length** of the **solid** portion of the graph.

Solution: The arclength of the graph is given by

$$\int_{\pi/3}^{5\pi/3}\sqrt{(r(\theta))^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta.$$

We have $dr/d\theta = -2\sin(\theta)$. This gives the answer below.

The arc length is $\int_{\pi/3}^{5\pi/3}\sqrt{(-1+2\cos(\theta))^2+(-2\sin(\theta))^2}\,d\theta$