5. [11 points] The parts of this question relate to the following polar graph, defined by the polar curve \( r(\theta) = -1 + 2\cos(\theta) \), on the domain \([0, 2\pi]\). Both the solid and dashed curves are part of the graph of \( r(\theta) \).

\[
\begin{array}{c}
\text{\( r(\theta) = -1 + 2\cos(\theta) \)} \\
\end{array}
\]

a. [2 points] What are all the angles \( \theta \), with \( 0 \leq \theta \leq 2\pi \), for which the graph passes through the origin?

\[\text{Solution:}\] The curve passes through the origin when \( r(\theta) = 0 \), so we need to solve \( \cos(\theta) = 1/2 \). This occurs for \( \theta \) in \([0, 2\pi]\) at the values \( \theta = \pi/3, 5\pi/3 \).

Answer(s): \( \theta = \pi/3, 5\pi/3 \)

b. [2 points] Determine the interval(s) within \([0, 2\pi]\) for which \( \theta \) traces out the dashed portion of the graph.

Answer(s): \([0, \pi/3]\) and \([5\pi/3, 2\pi]\)
c. [3 points] Write, but do not evaluate, an expression involving one or more integrals which gives the area enclosed by the dashed portion of the graph.

The area is \( \frac{1}{2} \int_{0}^{\pi/3} (-1 + 2 \cos(\theta))^2 \, d\theta + \frac{1}{2} \int_{\pi/3}^{2\pi} (-1 + 2 \cos(\theta))^2 \, d\theta \)

5. (continued) For your convenience, the polar graph referenced by this problem is reproduced here:

![Polar Graph]

d. [4 points] Write, but do not evaluate, an expression involving one or more integrals which gives the arc length of the solid portion of the graph.

**Solution:** The arclength of the graph is given by

\[
\int_{\pi/3}^{5\pi/3} \sqrt{\left( r(\theta) \right)^2 + \left( \frac{dr}{d\theta} \right)^2} \, d\theta.
\]

We have \( \frac{dr}{d\theta} = -2\sin(\theta) \). This gives the answer below.

The arc length is \( \int_{\pi/3}^{5\pi/3} \sqrt{(-1 + 2 \cos(\theta))^2 + (-2 \sin(\theta))^2} \, d\theta \)