6. [13 points] Some values of the function $f(x)$, its derivatives, and second derivatives are given in the table below. Assume for all positive integers n that $f^{(n)}(x)$ is continuous for all real numbers x.

x	-2	0	2	4	6
$f(x)$	1	2	0	1	2
$f'(x)$	3	2	1	0	-2
$f''(x)$	-3	-2	0	2	1

Using the information given above, find the following. Be sure to show all of your work. Your answers should not involve the letter f, but you do not need to simplify them.

a. [4 points] Find $\displaystyle\int_{-2}^{2} f'(x)f''(x)\,dx$.

Solution: We do a u-sub with $u = f'(x)$, so $du = f''(x)dx$. Therefore

$$\begin{aligned}\int_{-2}^{2} f'(x)f''(x)\,dx &= \int_{f'(-2)}^{f'(2)} u\,du \\ &= \int_{3}^{1} u\,du \\ &= \frac{u^2}{2}\Big|_3^1 \\ &= \frac{1}{2} - \frac{9}{2} = -4.\end{aligned}$$

Answer: -4

b. [3 points] Find the second degree polynomial that best approximates $f(x)$ near $x = 6$.

Solution: This is the second-degree Taylor polynomial of $f(x)$ near $x = 6$, which is $f(6) + f'(6)(x-6) + \frac{f''(6)}{2!}(x-6)^2$.

$f(x) \approx$ $2 - 2(x-6) + \frac{1}{2}(x-6)^2$

c. [3 points] Find $\displaystyle\lim_{x\to 0}\frac{f(x)-2+x^2}{x}$

Solution: Since $\lim_{x\to 0} f(x) = f(0)$ (as $f(x)$ is continuous) and $f(0) = 2$, this limit is in indeterminate form $0/0$, so we use L'Hopital's rule:

$$\lim_{x\to 0}\frac{f(x)-2+x^2}{x} \overset{LH}{=} \lim_{x\to 0}\frac{f'(x)+2x}{1} = 2.$$

The limit is **2**

d. [3 points] Find the approximate value of $\displaystyle\int_{-2}^{6} x^2 f(x)\,dx$ using MID(2).

Solution: The subintervals we use are $[-2,2]$ and $[2,6]$ which have midpoints 0 and 4, respectively. This means:

$$\text{MID}(2) = \Delta x\cdot(0^2 f(0) + 4^2 f(4)) = 4\cdot(16) = 64.$$

$\displaystyle\int_{-2}^{6} x^2 f(x)\,dx \approx$ **64**