2. [8 points] Consider the function $G(x) = x^3 \cos(2x)$.

a. [4 points] Give the first four nonzero terms of the Taylor series of $G(x)$ centered about $x = 0$.

Solution: Using the known Taylor series of $\cos(x)$ centered at $x = 0$, we have

$$x^3 \cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(2x)^{2n}x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!}x^{2n+3}.$$

Thus, the first four nonzero terms of the Taylor series of $G(x)$ about $x = 3$ are

$$x^3 - \frac{2^2}{2!}x^5 + \frac{2^4}{4!}x^7 - \frac{2^6}{6!}x^9.$$

Answer: $x^3 - \frac{2^2}{2!}x^5 + \frac{2^4}{4!}x^7 - \frac{2^6}{6!}x^9$

b. [4 points] Find $G^{(2023)}(0)$. You do not need to simplify.

Solution: The 2023rd power of x will appear in the Taylor series expansion of $G(x)$ with a nonzero coefficient. We can see this by noting that $2n + 3 = 2023$ if $n = 1010$. Therefore,

$$\frac{G^{(2023)}(0)}{2023!} = \frac{(-1)^{1010}\, 2^{2\cdot 1010}}{(2\cdot 1010)!},$$

so $G^{(2023)}(0) = (2023)! \cdot \frac{(-1)^{1010}\, 2^{2020}}{2020!}$.

Answer: $(2023)! \cdot \frac{(-1)^{1010}\, 2^{2020}}{2020!}$