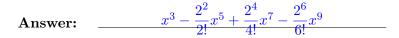
- **2**. [8 points] Consider the function $G(x) = x^3 \cos(2x)$.
 - **a**. [4 points] Give the first four nonzero terms of the Taylor series of G(x) centered about x = 0.

Solution: Using the known Taylor series of cos(x) centered at x = 0, we have

$$x^{3}\cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^{n}}{(2n)!} (2x)^{2n} x^{3} = \sum_{n=0}^{\infty} \frac{(-1)^{n} 2^{2n}}{(2n)!} x^{2n+3}.$$

Thus, the first four nonzero terms of the Taylor series of G(x) about x = 3 are

$$x^{3} - \frac{2^{2}}{2!}x^{5} + \frac{2^{4}}{4!}x^{7} - \frac{2^{6}}{6!}x^{9}.$$



b. [4 points] Find $G^{(2023)}(0)$. You do not need to simplify.

Solution: The 2023rd power of x will appear in the Taylor series expansion of G(x) with a nonzero coefficient. We can see this by noting that 2n + 3 = 2023 if n = 1010. Therefore, $G(2023)(0) = (-1)^{1010} c^{2} c^{1010}$

$$\frac{G^{(2023)}(0)}{2023!} = \frac{(-1)^{1010} 2^{2\cdot1010}}{(2\cdot1010)!},$$

so $G^{(2023)}(0) = (2023)! \cdot \frac{(-1)^{1010} 2^{2020}}{2020!}.$

A	$(2023)1$, $(-1)^{1010}2^{2020}$
Answer:	(2023): •
	2020!