2. [8 points] Consider the function \( G(x) = x^3 \cos(2x) \).

   a. [4 points] Give the first four nonzero terms of the Taylor series of \( G(x) \) centered about \( x = 0 \).

   Solution: Using the known Taylor series of \( \cos(x) \) centered at \( x = 0 \), we have
   \[
   x^3 \cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!)} (2x)^{2n} x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n2^{2n}}{(2n)!} x^{2n+3}.
   \]
   Thus, the first four nonzero terms of the Taylor series of \( G(x) \) about \( x = 3 \) are
   \[
   x^3 - \frac{2^2}{2!} x^5 + \frac{2^4}{4!} x^7 - \frac{2^6}{6!} x^9.
   \]

   Answer: \( x^3 - \frac{2^2}{2!} x^5 + \frac{2^4}{4!} x^7 - \frac{2^6}{6!} x^9 \)

   b. [4 points] Find \( G^{(2023)}(0) \). You do not need to simplify.

   Solution: The 2023rd power of \( x \) will appear in the Taylor series expansion of \( G(x) \) with a nonzero coefficient. We can see this by noting that \( 2n + 3 = 2023 \) if \( n = 1010 \). Therefore,
   \[
   G^{(2023)}(0) = \frac{(-1)^{1010} \cdot 2^{1010}}{(2 \cdot 1010)!},
   \]
   so \( G^{(2023)}(0) = (2023)! \cdot \frac{(-1)^{1010} \cdot 2^{2020}}{2020!} \).

   Answer: \( (2023)! \cdot \frac{(-1)^{1010} \cdot 2^{2020}}{2020!} \)