3. [12 points] Antonia the ant is entering her first bug race. The track runs from the start line at the south end, represented by $y = 0$, to the finish line at the north end, represented by $y = 4$. All distances are given in feet.
Antonia's position t seconds after the race begins is given in parametric equations by:

$$x = \sin\left(\frac{\pi t}{2}\right), \qquad y = 1.5^t - 1,$$

a. [2 points] What is Antonia's position 2 seconds into the race?

Solution: To find this, we plug $t = 2$ into the equations given for the x and y coordinates of Antonia. We find $x = \sin(\pi) = 0$ and $y = 1.5^2 - 1 = 1.25$.

$x =$ 0 $\qquad y =$ 1.25

b. [3 points] At what time does Antonia reach the finish line?

Solution: We set $y = 4$ to to get $1.5^t - 1 = 4$. Therefore, $1.5^t = 5$, and so $t = \dfrac{\ln(5)}{\ln(1.5)}$.

The time is $t = \dfrac{\ln(5)}{\ln(1.5)}$

c. [3 points] What is the first time during the race that Antonia is travelling directly north?

Solution: Note that y is always increasing, so we only need to find the first time that $\dfrac{dx}{dt} = 0$.
We have $\dfrac{dx}{dt} = \dfrac{\pi}{2}\cos\left(\dfrac{\pi t}{2}\right)$, which is equal to 0 when $t = 1$.

The time is $t =$ 1

d. [4 points] Write an expression involving one or more integrals that gives the total distance, in feet, that Antonia traveled during the race. Do not evaluate your integral(s).

Solution: We use the arclength formula, with $\dfrac{dx}{dt} = \dfrac{\pi}{2}\cos\left(\dfrac{\pi t}{2}\right)$, $\dfrac{dy}{dt} = \ln(1.5)1.5^t$, and using the time we found in part b.

The distance is $\displaystyle\int_0^{\ln(5)/\ln(1.5)} \sqrt{\left(\frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right)\right)^2 + (\ln(1.5)1.5^t)^2}\, dt$