5. [7 points] A local beet company, Dope Beets Inc., is developing a new beet with an adjustable growth rate for its many different customers. The growth rate of their new beet, measured in pounds per day, t days after a beet is planted, is given by

$$r(t) = \frac{5t^2}{t^k + t + 1},$$

for some adjustable value $k > 1$.

a. [4 points] Suppose a new beet initially weighs 2 pounds. Write an expression involving an integral for the weight, in pounds, of the beet t days after it is planted.

Answer: $2 + \displaystyle\int_0^t \frac{5x^2}{x^k + x + 1}\,\mathrm{d}x$

b. [3 points] Dope Beets Inc. wants to adjust the value of k such that a planted beet will never have infinite weight, even if the beet is allowed to grow forever. Which values of k would keep the weight finite? Give your answer as a value, list of values, or interval, as appropriate. No justification is required.

Solution: (Not required). Consider the integral

$$\int_1^\infty \frac{1}{x^{k-2}}\,\mathrm{d}x.$$

Let $p = k - 2$; this integral converges by the p-test when $p > 1$, or when $k > 3$. By (properly) using the direct or limit comparison tests, this shows the $\displaystyle\int_0^\infty \frac{5x^2}{x^k + x + 1}\,\mathrm{d}x$ (the total change in a beet's weight over all time) converges to a finite value.

Answer: $k > 3$