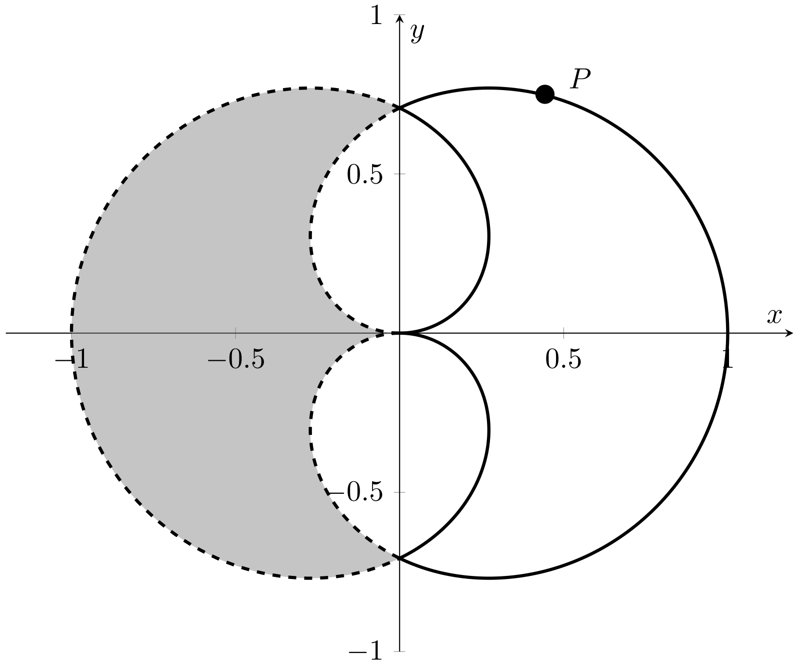
7. [16 points] A particle moves along a path given by the polar curve $r = \cos(\theta/2)$, $0 \le \theta \le 4\pi$. The polar curve is graphed below. A portion of the polar curve is dashed.



a. [4 points] The distance from the origin to the point labeled P is $\sqrt{3}/2$. Find the Cartesian coordinates corresponding to the point labeled P.

Solution: Consider the polar coordinates (r, θ) of the point P. By the hypothesis of the problem, $r = \sqrt{3}/2$. To determine θ, we want to solve $\cos(\theta/2) = \sqrt{3}/2$. We find that

$$\theta/2 = \pi/6 \Rightarrow \theta = \pi/3.$$

Plugging the polar coordinates into $x = r\cos(\theta), y = r\sin(\theta)$ gets us

$$x = \frac{\sqrt{3}}{2}\cos(\pi/3) = \frac{\sqrt{3}}{4}, \quad y = \frac{\sqrt{3}}{2}\sin(\pi/3) = \frac{3}{4}$$

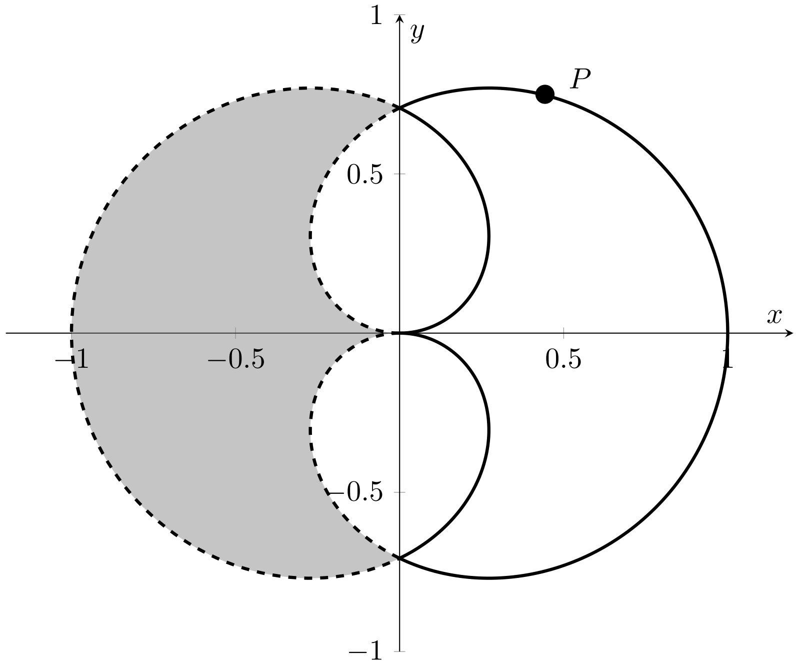
$(x, y) =$ $(\sqrt{3}/4, 3/4)$

b. [4 points] For what values of θ in $[0, 4\pi]$ does the particle pass through the origin?

Solution: We solve $\cos(\theta/2) = 0$ and see that $\theta/2 = m\pi/2$, where m is an odd integer. The only θ in the interval $[0, 4\pi]$ that satisfy this equation are $\theta = \pi, 3\pi$.

$\theta =$ $\pi, 3\pi$

7. (continued) The graph of the polar curve $r = \cos(\theta/2)$, with $0 \le \theta \le 4\pi$, from the previous page is reproduced below:



c. [4 points] Determine the interval(s) within $[0, 4\pi]$ for which θ traces out the **dashed** portion of the graph.

Answer: $[\pi/2, \pi] \cup [3\pi/2, 5\pi/2] \cup [3\pi, 7\pi/2]$

d. [4 points] Write an expression involving one or more integrals for the shaded area enclosed by the dashed portion of the particle's path. Do not evaluate your integral(s).

Answer: $\displaystyle\int_{3\pi/2}^{5\pi/2} \frac{1}{2}\cos^2(\theta/2)\,\mathrm{d}\theta - 2\int_{\pi/2}^{\pi} \frac{1}{2}\cos^2(\theta/2)\,\mathrm{d}\theta$