9. [6 points] Let $f(x)$ and $g(x)$ be two continuous and differentiable functions on $[1, \infty)$. Further, suppose these functions have the following properties:

- $F(x) = x(g(x) + 1)$ is an antiderivative of $f(x)$ for $x \geq 1$,
- $g(1) = 10$,
- $\lim_{x \to \infty} g(x) = -1$,
- $\lim_{x \to \infty} x^2 g'(x) = 17$.

Compute the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_1^\infty f(x)\,\mathrm{d}x$$

Circle one: **Diverges** **Converges to** _____