

1. [11 points] Consider the polar curve $r = \theta \sin \theta$.

- a. [2 points] What are the x - and y -coordinates of the curve in terms of θ ? Use this to write a set of parametric equations for the curve.

Answer: $x(\theta) = \underline{\theta \sin \theta \cos \theta}$ and $y(\theta) = \underline{\theta \sin^2 \theta}$

- b. [2 points] Which of the following points are on the curve $r = \theta \sin \theta$? Circle **all** options which apply.

i. $\theta = \frac{\pi}{2}, r = \frac{\pi}{2}$

v. $x = 0, y = \frac{\pi}{2}$

ii. $\theta = \frac{3\pi}{2}, r = \frac{3\pi}{2}$

vi. $x = 0, y = -\frac{\pi}{2}$

iii. $\theta = \pi, r = \pi$

vii. $x = 0, y = -\frac{3\pi}{2}$

iv. $\theta = 2\pi + \frac{\pi}{2}, r = \frac{\pi}{2}$

viii. NONE OF THESE

- c. [1 point] Find $\frac{dy}{d\theta}$ in terms of θ .

Solution:

$$\begin{aligned} \frac{dy}{d\theta} &= \frac{d}{d\theta} \theta \sin^2 \theta \\ &= \sin^2 \theta + \theta(2 \sin \theta \cos \theta) \\ &= \sin \theta(\sin \theta + 2\theta \cos \theta) \end{aligned}$$

Answer: $\frac{dy}{d\theta} = \underline{\sin \theta(\sin \theta + 2\theta \cos \theta)}$

- d. [2 points] At which of the following values of θ could the curve $r = \theta \sin \theta$ have a horizontal tangent line? Circle **all** options which apply.

i. $\theta = \frac{\pi}{2}$

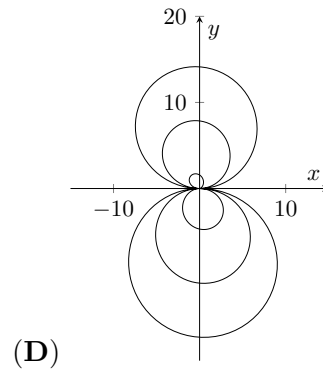
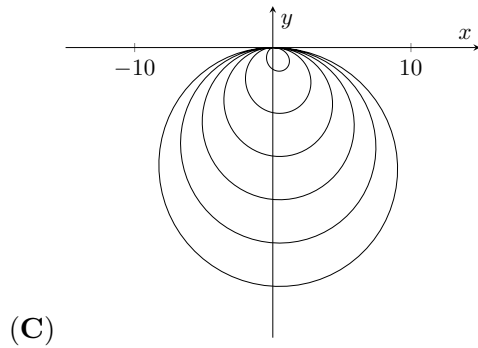
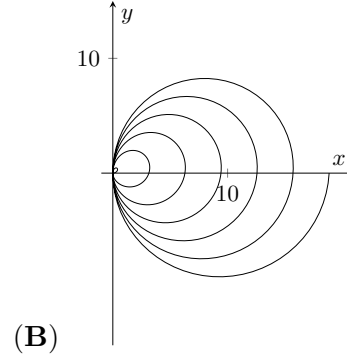
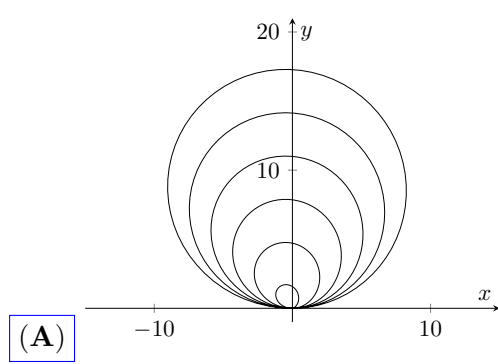
iii. $\theta = 2024\pi$

ii. $\theta = \pi$

iv. $\theta = \frac{2025}{2}\pi$

v. NONE OF THESE

- e. [2 points] Which of the following could be the graph of the polar curve $r = \theta \sin \theta$, with $0 \leq \theta \leq 6\pi$? Circle the **one** best option.



- f. [2 points] Which of the following integrals gives the length of the curve $r = \theta \sin \theta$, for $0 \leq \theta \leq 6\pi$? Circle **all** options which apply.

i. $\int_0^{6\pi} \sqrt{(\theta (\cos^2 \theta - \sin^2 \theta) + \sin \theta \cos \theta)^2 + (\sin \theta (2\theta \cos \theta + \sin \theta))^2} d\theta$

ii. $\int_0^{6\pi} \sqrt{(\theta \cos \theta + \sin \theta)^2 + (\theta \sin \theta)^2} d\theta$

iii. $\int_0^{2\pi} \sqrt{(\theta (\cos^2 \theta - \sin^2 \theta) + \sin \theta \cos \theta)^2 + (\sin \theta (2\theta \cos \theta + \sin \theta))^2} d\theta$

iv. $\int_0^{2\pi} \sqrt{(\theta \cos \theta + \sin \theta)^2 + (\theta \sin \theta)^2} d\theta$

v. NONE OF THE ABOVE.