3. [13 points] In this week's snail race there are three snail competitors: snails A, B and C. All three snails start at the left wall of an aquarium (located at $x = 0$), and must cross the tank to the right-hand wall (located at $x = 100$). The paths of snails A and B are given below. All three snails start at $t = 0$, and stop when they reach the right-hand wall of the aquarium. The time t is measured in seconds, and all distances are in millimeters.

$$\begin{aligned} \text{A}: &\quad x(t) = 10t, &\quad y(t) &= \frac{t^2}{3} \\ \text{B}: &\quad x(t) = t^2 - 7t + 82, &\quad y(t) &= -\frac{t^3}{4} + 250 \end{aligned}$$

a. [3 points] When do snails A and B finish the race? Which of these two snails reaches the finish line first?

Solution: Each snail finishes the race when their x-coordinate is 100. For snail A this occurs at $t = 10$. For snail B it occurs when $t^2 - 7t + 82 = 100$, i.e., when $(t - 9)(t + 2) = 0$, so $t = 9$.

Answer: Snail A finishes at $t =$ **10** seconds.

Answer: Snail B finishes at $t =$ **9** seconds.

Answer: The winner of these snails is snail **B** .

b. [4 points] Write an expression using one or more integrals for the distance that snail A travels during the race. Do not evaluate any integrals in your expression.

Solution: The distance snail A travels is given by

$$\int_0^{10} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{10} \sqrt{(10)^2 + \left(\frac{2t}{3}\right)^2}\, dt = \int_0^{10} \sqrt{100 + \frac{4t^2}{9}}\, dt$$

Answer: Snail A travels $\int_0^{10} \sqrt{100 + \frac{4t^2}{9}}\, dt$ mm.

c. [3 points] Snail C travels in a straight line from the origin through the point $(40, 30)$. Which, if any, of the following could be a parametric equation describing snail C's *path* (disregarding speed) during the race? Circle **all** options which apply.

i. $x(t) = 40t, \quad y(t) = 30t$

ii. $x(t) = 30t, \quad y(t) = 40t$

iii. $x(t) = \sin t, \quad y(t) = \frac{3}{4}\sin t$

iv. $x(t) = 30\sin t, \quad y(t) = 40\sin t$

v. $x(t) = 40\sin(t + \frac{\pi}{2}), \quad y(t) = 30\sin(t + \frac{\pi}{2})$

vi. None of the above

d. [3 points] Now assume that snail C travels at a constant speed of 10mm/s, still in a straight line from the origin through the point $(40, 30)$. Which, if any, of the following could be a parametric equation describing snail C's *motion* (including speed) during the race? Circle **all** options which apply.

i. $x(t) = 8t, \quad y(t) = 6t$

ii. $x(t) = 40t, \quad y(t) = 30t$

iii. $x(t) = 8\sin t, \quad y(t) = 6\sin t$

iv. $x(t) = 40\sin t, \quad y(t) = 30\sin t$

v. $x(t) = 10\sin t, \quad y(t) = 10\cos t$

vi. None of the above