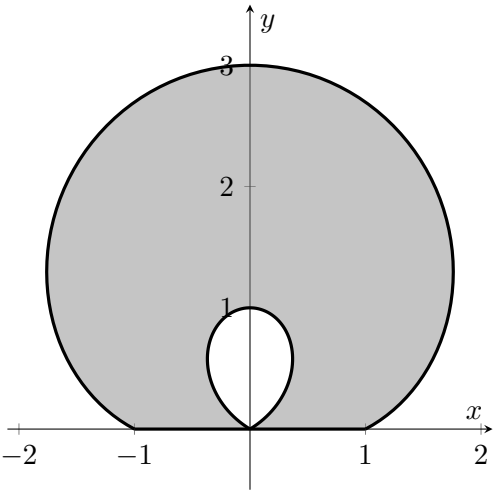
4. [14 points]

Delema Inventions Inc is coming up with a design for a window. The window has the shape shown to the right, formed from the portion of the polar curve $r(\theta) = 1+2\sin\theta$ with $y \geq 0$.
The outer loop (shaded) is made of green glass, and the inner loop (unshaded) is made of blue glass. The perimeter (including the perimeter of the inner loop, and the base along the x-axis) is lined with a black material.



a. [4 points] There are four values of θ with $0 \leq \theta < 2\pi$ such that $y = 0$ for the polar curve $r(\theta) = 1 + 2\sin\theta$. Find all four values.

Solution: We have $y(\theta) = (1+2\sin\theta)\sin\theta$, so $y(\theta) = 0$ exactly where $\sin\theta = 0$ or $1+2\sin\theta = 0$. This happens when $\theta = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$.

Answer: $\theta =$ $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

b. [5 points] Find an expression involving one or more integrals for the length of black material Delema Inventions Inc will need to build the window. Remember that this material lines the edge of the outer loop, the edge of the inner loop, and also the base along the x-axis. Do not evaluate your integral(s).

Solution: Let $f(\theta) = 1 + 2\sin\theta$. Then $f'(\theta) = 2\cos\theta$.
The outer loop is traced out as θ ranges from 0 to π, the inner loop is traced out as θ ranges from $\frac{7\pi}{6}$ to $\frac{11\pi}{6}$, and the base has length 2. Altogether, the length of the black material is then:

$$2 + \int_0^{\pi} \sqrt{(1+2\sin\theta)^2 + (2\cos\theta)^2}\,d\theta + \int_{7\pi/6}^{11\pi/6} \sqrt{(1+2\sin\theta)^2 + (2\cos\theta)^2}\,d\theta.$$

Answer: $2 + \int_0^{\pi} \sqrt{(1+2\sin\theta)^2 + (2\cos\theta)^2}\,d\theta + \int_{7\pi/6}^{11\pi/6} \sqrt{(1+2\sin\theta)^2 + (2\cos\theta)^2}\,d\theta.$

c. [5 points] Find an expression involving one or more integrals for the area of green glass (the shaded region) which Delema Inventions Inc will need to build the window. Do not evaluate your integral(s).

Solution: The area of green glass is the area enclosed by the outer loop minus the area enclosed by the inner loop, which is

$$\frac{1}{2}\int_0^{\pi} (1+2\sin\theta)^2\,d\theta - \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\,d\theta.$$

Answer: $\frac{1}{2}\int_0^{\pi} (1+2\sin\theta)^2\,d\theta - \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\,d\theta.$