5. [13 points]

a. [2 points] Find $\int \frac{x}{x+1}\,dx$, showing all of your work.

Solution: Re-writing the numerator we get:

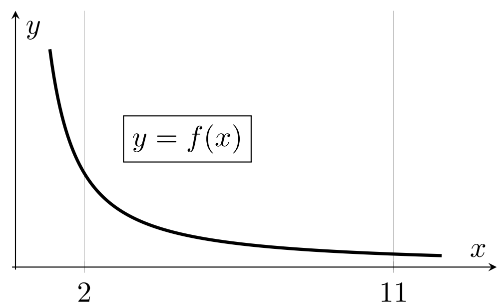
$$\begin{aligned}\int \frac{x}{x+1}\,dx &= \int \frac{x+1-1}{x+1}\,dx \\ &= \int 1 - \frac{1}{x+1}\,dx \\ &= x - \ln|x+1| + C\end{aligned}$$

Answer: $x - \ln|x+1| + C$

Now consider the region in the first quadrant bounded by the x-axis, the lines $x = 2$ and $x = 11$, and the curve

$$f(x) = \frac{5x+12}{x(x+1)},$$

as shown in the figure to the right.



b. [5 points] Find an expression involving one or more integrals for the volume of the solid of revolution given by rotating this region around the y-axis. Your expression should not contain the letter f. Do not evaluate your integral(s).

Solution: We use vertical slices, which gives rise to cylinders. For this region, x ranges between 2 and 11 so we get

$$\int_2^{11} 2\pi x f(x)\,dx = 2\pi \int_2^{11} \frac{5x+12}{x+1}\,dx$$

Answer: $2\pi \int_2^{11} \frac{5x+12}{x+1}\,dx$

c. [4 points] Find a LEFT(3) approximation to your integral from part (b). Write out all the terms in your sum. You do not need to simplify, but your final answer should not involve the letter f.

Solution: Each rectangle has width 3, so LEFT(3) is

$$2\pi(3)\left(\frac{5(2)+12}{2+1} + \frac{5(5)+12}{5+1} + \frac{5(8)+12}{8+1}\right)$$

Answer: $2\pi(3)\left(\frac{5(2)+12}{2+1} + \frac{5(5)+12}{5+1} + \frac{5(8)+12}{8+1}\right)$

d. [2 points] Evaluate your expression from part (b). You may use your answer from (a).

Solution: Using our solution to part (a) we get:

$$\begin{aligned} 2\pi\int_2^{11}\frac{5x+12}{x+1}\,dx &= 2\pi\left(5\int_2^{11}\frac{x}{x+1}\,dx+12\int_2^{11}\frac{1}{x+1}\,dx\right)\\ &= 2\pi\left(5(x-\ln|x+1|)+12\ln|x+1|\right)_2^{11}\\ &= 2\pi\left(5x+7\ln|x+1|\right)_2^{11}\\ &= 2\pi\left(5(11)+7\ln(12)-5(2)-7\ln(3)\right)\\ &= 2\pi\left(45+7\ln(4)\right) \end{aligned}$$

Answer: $2\pi\left(5(11)+7\ln(12)-5(2)-7\ln(3)\right)$