10. [13 points] The parts of this problem are unrelated. No justification is required for your answers.

a. [4 points] Suppose that $f(x)$ is a positive, decreasing, differentiable function which is defined for all real numbers, and that for $n \geq 1$, a_n is given by the LEFT(n) approximation to the integral $\int_1^{n+1} f(x)\ dx$. If $\lim\limits_{n \to \infty} a_n = 23$, then which of the following must be true? Circle **all** options which apply.

i. $\displaystyle\int_1^\infty f(x)\ dx$ converges to 23.

v. $\displaystyle\sum_{n=1}^\infty f(n)$ converges to 23.

ii. $\displaystyle\int_1^\infty f(x)\ dx$ converges to a number greater than 23.

vi. $\displaystyle\sum_{n=1}^\infty f(n)$ converges to a number greater than 23.

iii. $\displaystyle\int_1^\infty f(x)\ dx$ converges to a number less than 23.

vii. $\displaystyle\sum_{n=1}^\infty f(n)$ converges to a number less than 23.

iv. $\displaystyle\int_1^\infty f(x)\ dx$ could diverge.

viii. $\displaystyle\sum_{n=1}^\infty f(n)$ could diverge.

b. [2 points] Suppose that $f(h)$ is a probability density function for h, the maximum depth, in hundreds of meters, that a king penguin reaches on a single dive to catch fish. Which of the following statements is best supported by the equation $f(1) = 0.5$? Circle the **one** best answer.

i. There is about a 50% chance that a king penguin dives to maximum depth of 100 meters in any given dive.

ii. There is roughly a 100% chance that a king penguin will dive to a maximum depth of less than 50 meters in any given dive.

iii. If a king penguin were to dive for fish 200 times, the maximum depth of about 20 of those dives would be between 100 and 120 meters.

iv. Approximately 0.5% of all of a king penguin's dives will be to a maximum depth of between 50 and 150 meters.

v. The median maximum depth that king penguins dive to is 100 meters.

vi. NONE OF THESE

c. [4 points] Which of the following series converge? Circle **all** options that apply.

i. $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n}{2n+1}$

iii. $\displaystyle\sum_{n=1}^\infty \frac{5^n}{n!}$

v. $\displaystyle\sum_{n=3}^\infty \frac{1}{n\ln(n)}$

ii. $\displaystyle\sum_{n=1}^\infty \frac{\cos(n)}{n^3}$

iv. $\displaystyle\sum_{n=1}^\infty \frac{(-1)^{2n}}{n^{2/3}}$

vi. NONE OF THESE

d. [3 points] Which of the following series converge to 50? Circle **all** options that apply.

i. $\displaystyle\sum_{n=0}^\infty \frac{(-1)^n\, 50}{(2n)!} \left(\frac{\pi}{2}\right)^{2n}$

iii. $\displaystyle\sum_{n=0}^\infty \frac{(\ln(50))^n}{n!}$

v. NONE OF THESE

ii. $\displaystyle\sum_{n=0}^\infty \frac{(-1)^n\, 50}{(2n+1)!} \left(\frac{\pi}{2}\right)^{2n+1}$

iv. $\displaystyle\sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} \left(e^{50} - 1\right)^n$