

## 10. [13 points]

a. [4 points] Suppose that  $f(x)$  is a positive, decreasing, differentiable function which is defined for all real numbers, and that for  $n \geq 1$ ,  $a_n$  is given by the LEFT( $n$ ) approximation to the integral  $\int_1^{n+1} f(x) dx$ . If  $\lim_{n \rightarrow \infty} a_n = 23$ , then which of the following must be true? Circle **all** options which apply.

i.  $\int_1^{\infty} f(x) dx$  converges to 23.

v.  $\sum_{n=1}^{\infty} f(n)$  converges to 23.

ii.  $\int_1^{\infty} f(x) dx$  converges to a number greater than 23.

vi.  $\sum_{n=1}^{\infty} f(n)$  converges to a number greater than 23.

iii.  $\int_1^{\infty} f(x) dx$  converges to a number less than 23.

vii.  $\sum_{n=1}^{\infty} f(n)$  converges to a number less than 23.

iv.  $\int_1^{\infty} f(x) dx$  could diverge.

viii.  $\sum_{n=1}^{\infty} f(n)$  could diverge.

b. [2 points] Suppose that  $f(h)$  is a probability density function for  $h$ , the maximum depth, in hundreds of meters, that a king penguin reaches on a single dive to catch fish. Which of the following statements is best supported by the equation  $f(1) = 0.5$ ? Circle the **one** best answer.

i. There is about a 50% chance that a king penguin dives to maximum depth of 100 meters in any given dive.

ii. There is roughly a 100% chance that a king penguin will dive to a maximum depth of less than 50 meters in any given dive.

iii. If a king penguin were to dive for fish 200 times, the maximum depth of about 20 of those dives would be between 100 and 120 meters.

iv. Approximately 0.5% of all of a king penguin's dives will be to a maximum depth of between 50 and 150 meters.

v. The median maximum depth that king penguins dive to is 100 meters.

vi. NONE OF THESE

c. [4 points] Which of the following series converge? Circle **all** options that apply.

i.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$

iii.  $\sum_{n=1}^{\infty} \frac{5^n}{n!}$

v.  $\sum_{n=3}^{\infty} \frac{1}{n \ln(n)}$

ii.  $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}$

iv.  $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^{2/3}}$

vi. NONE OF THESE

d. [3 points] Which of the following series converge to 50? Circle **all** options that apply.

i.  $\sum_{n=0}^{\infty} \frac{(-1)^n 50}{(2n)!} \left(\frac{\pi}{2}\right)^{2n}$

iii.  $\sum_{n=0}^{\infty} \frac{(\ln(50))^n}{n!}$

v. NONE OF THESE

ii.  $\sum_{n=0}^{\infty} \frac{(-1)^n 50}{(2n+1)!} \left(\frac{\pi}{2}\right)^{2n+1}$

iv.  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (e^{50} - 1)^n$