3. [5 points] Throughout this problem, suppose that:

- a_n is a sequence with $a_1 = 60$, and with $a_{n+1} = \frac{1}{3}a_n$ for all $n \geq 1$.
- b_n is a sequence with $b_1 = \frac{1}{2}$, and with $b_{n+1} = 2b_n$ for all $n \geq 1$.
- $S_n = \sum_{j=1}^{n} a_j$ and $R_n = \sum_{k=1}^{n} S_k$.

For each of the following sequences, determine whether the sequence converges or diverges, and if it converges, determine the value that it converges to. Justification is not required.

a. [1 point] a_n

Solution: Since $a_{n+1} = \frac{1}{3}a_n$, we see that $a_n = 60\left(\frac{1}{3}\right)^{n-1}$ and thus $\lim_{n\to\infty} a_n = 0$.

Circle one: **Diverges** $\boxed{\textbf{Converges to } \underline{\quad 0 \quad}}$

b. [1 point] b_n

Solution: Using a similar argument as **a.** we have, $b_n = \frac{2^{n-1}}{2}$, from which we can see that b_n diverges.

Circle one: $\boxed{\textbf{Diverges}}$ **Converges to** ______________

c. [1 point] $c_n = a_n \cdot b_n$

Solution: Using **a.** and **b.** we can compute that $c_n = a_n \cdot b_n = 30\left(\frac{2}{3}\right)^{n-1}$ and thus $\lim_{n\to\infty} c_n = 0$.

Circle one: **Diverges** $\boxed{\textbf{Converges to } \underline{\quad 0 \quad}}$

d. [1 point] S_n

Solution: Note that S_n is a finite geometric series, and so

$$\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} 60\left(\frac{1}{3}\right)^{n-1} = 60\left(\frac{1}{1-\frac{1}{3}}\right) = 90.$$

Circle one: **Diverges** $\boxed{\textbf{Converges to } \underline{\quad 90 \quad}}$

e. [1 point] R_n

Solution: Since $R_n = \sum_{k=1}^{n} S_k$ and $\lim_{n\to\infty} S_n = 90 \neq 0$, the nth term test for divergence tells us that R_n diverges.

Circle one: $\boxed{\textbf{Diverges}}$ **Converges to** ______________