4. [13 points] A vertical climbing wall is described in coordinates, where (x, y) is the position x meters to the right of a central podium and y meters above the floor. Suppose two climbers, Sara and Tina, begin climbing the wall at the same time. Their positions t minutes after they start climbing are given by:

$$\text{Sara: } \begin{cases} x(t) = -(t-1)^2 \\ y(t) = 3t, \end{cases} \qquad \text{Tina: } \begin{cases} x(t) = 1 + \cos(\pi t) \\ y(t) = t^2 + 2. \end{cases}$$

a. [4 points] Tina and Sara bumped into each other one time during the process of climbing. At what time did this happen? Justify your answer.

Solution: They bump into each other if their x and y-coordinates are both the same. If we equate the y coordinates, we get $3t = t^2 + 2$ and solving gives us $t = 1$ and $t = 2$.
Plugging in $t = 1$ and $t = 2$ to the x-coordinate parameterizations we note that the x coordinates of Sara and Tina are the same at $t = 1$. Therefore, they bumped into each other at $t = 1$.

Answer: $t =$ 1

b. [2 points] After Sara and Tina bumped into each other, they climbed for some further distance and eventually crossed the finish line, which is a horizontal line, at the same time. When did they cross the finish line, and what is the equation of the finish line?

Solution: Crossing the finish line at the same time means that Sara and Tina's y-coordinates are the same at that time. From part **a.**, we know this happens at $t = 1$ and $t = 2$. We already know from **a.** that they bumped into each other at $t = 1$, so $t = 2$ must be the time at which they both crossed the finish line. We plug in $t = 2$ to either Sara or Tina's y-coordinates and get $y = 6$.

Answer: They crossed the finish line at $t =$ 2

Answer: The equation of the finish line is $y =$ 6

c. [4 points] Write an expression involving one or more integrals that gives the total distance traveled by Tina from the time she starting climbing until she crossed the finish line. Do not evaluate your integral(s). Show your work.

Solution: For Tina, $x'(t) = -\pi \sin(\pi t)$ and $y'(t) = 2t$, so the total distance traveled is

$$\int_0^2 \sqrt{x'(t)^2 + y'(t)^2}\, dt = \int_0^2 \sqrt{\pi^2 \sin^2(\pi t) + 4t^2}\, dt.$$

Answer: $\displaystyle\int_0^2 \sqrt{\pi^2 \sin^2(\pi t) + 4t^2}\, dt$

d. [3 points] What is Tina's speed at $t = \dfrac{1}{2}$? **Include units**. You do not need to simplify your answer but you should show your work.

Solution: Tina's speed at time t is $\sqrt{\pi^2 \sin^2(\pi t) + 4t^2}$ m/min, so plugging in $t = \dfrac{1}{2}$ we get $\sqrt{\pi^2 + 1}$ m/min .

Answer: $\sqrt{\pi^2 + 1}$ **Units:** m/min