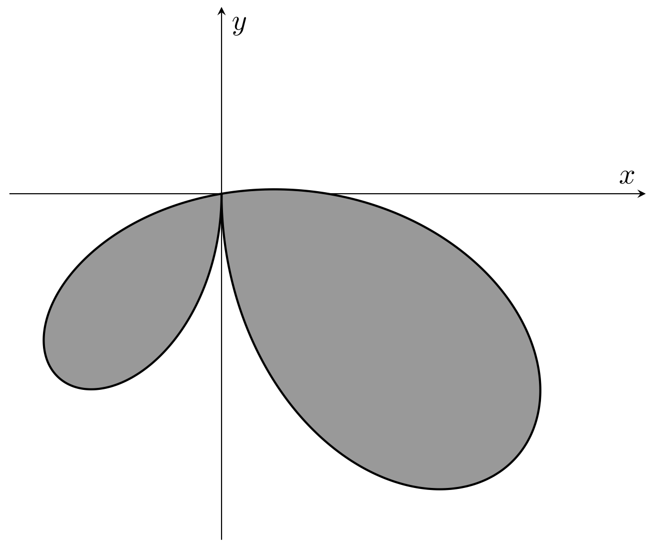
6. [6 points] A graphic designer makes posters for David, a singer who famously used to have a unique haircut. In a poster for an old album, the boundary of David's haircut is given by the polar curve $r(\theta) = \cos(\theta) + 3\sin(2\theta)$, where $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$. A graph of the curve is shown below, and David's hair is represented by the shaded region.



a. [2 points] What are the x- and y-coordinates of the curve in terms of θ?

Solution: We have

$$x(\theta) = r(\theta)\cos(\theta) = \cos^2(\theta) + 3\sin(2\theta)\cos(\theta)$$
$$y(\theta) = r(\theta)\sin(\theta) = \cos(\theta)\sin(\theta) + 3\sin(2\theta)\sin(\theta)$$

Answer: $x(\theta) =$ $\cos^2(\theta) + 3\sin(2\theta)\cos\theta$

Answer: $y(\theta) =$ $\cos(\theta)\sin(\theta) + 3\sin(2\theta)\sin(\theta)$

b. [2 points] Find $\frac{dx}{d\theta}$ in terms of θ.

Solution: Using chain rule and product rule, we have

$$\frac{dx}{d\theta} = -2\cos(\theta)\sin(\theta) + 6\cos(2\theta)\cos(\theta) - 3\sin(2\theta)\sin(\theta).$$

Answer: $\frac{dx}{d\theta} =$ $-2\cos(\theta)\sin(\theta) + 6\cos(2\theta)\cos(\theta) - 3\sin(2\theta)\sin(\theta)$

c. [2 points] At which of the following values of θ could the above curve have a vertical tangent line? Circle all options that apply.

Solution: We look at values of θ such that $\frac{dx}{d\theta} = 0$, and can see that $\theta = \pi/2$ and $\theta = 3\pi/2$ satisfy the requirement.

i. $\theta = \pi/2$

ii. $\theta = \pi$

iii. $\theta = 3\pi/2$

iv. None of the above