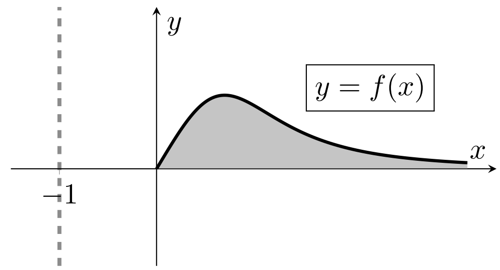
8. [9 points] The parts of this problem are unrelated.

a. [4 points]
Consider the infinite region in the first quadrant between the x-axis and the curve $y = f(x)$ where

$$f(x) = \frac{\arctan(x)}{x^3+1},$$

as shaded in the figure to the right.

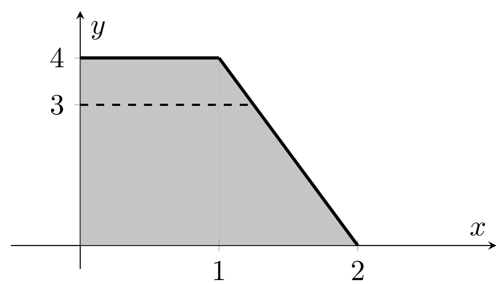


Find an expression involving one or more integrals for the volume of the solid of revolution given by rotating the infinite shaded region around the line $x = -1$. Your expression should not contain the letter f. Do not evaluate your integral(s).

Solution: Using the shell method, we see that the volume is

$$\int_0^\infty 2\pi(x+1)f(x)\ dx = \int_0^\infty 2\pi(x+1)\frac{\arctan(x)}{x^3+1}\ dx.$$

b. [5 points]
Consider the region in the first quadrant bounded by the y-axis, the x-axis, the line $y = 4$, and the line $y = 8 - 4x$, as shaded in the figure to the right.
Assume that all distances are in meters. You may assume that the acceleration due to gravity is $g = 9.8\text{m/s}^2$.



A tank is formed by rotating this region around the y-axis. The tank is partially filled with a liquid, to a depth of 3 meters from the bottom of the tank (indicated by the dashed line in the figure above). Suppose the liquid has density $p(y) = (1+y)^{1/3}$ kg/m^3 where y, in meters, is the height of a point above the bottom of the tank. Find an expression involving one or more integrals for the work, in Joules, needed to pump all the liquid to the top of the tank. Your answer should not involve the letter p. Do not evaluate your integral(s).

Solution: If we consider a horizontal slice at height y from the bottom having thickness Δy, the horizontal distance from the y axis is $x = \dfrac{8-y}{4}$ m, so the volume of that slice is

$$\pi\left(\frac{8-y}{4}\right)^2\Delta y \quad \text{m}^3, \quad \text{so its weight is} \quad 9.8\pi\left(\frac{8-y}{4}\right)^2(1+y)^{1/3}\Delta y \quad \text{kg}.$$

Therefore the work required to pull the slice out of the tank is

$$9.8\pi\left(\frac{8-y}{4}\right)^2(1+y)^{1/3}(4-y)\Delta y \quad \text{Joules},$$

and the total amount of work is

$$\int_0^3 9.8\pi\left(\frac{8-y}{4}\right)^2(1+y)^{1/3}(4-y)\ dy \quad \text{Joules}.$$