9. (6 pts) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. How much volume is enclosed by all the chambers? Assume for simplicity that there are infinitely many chambers. Show your work.

Because the chambers grow by a constant factor each time, they form a geometric series. If each is 20% larger than the previous, then the ratio between them is 1.2. But this is the ratio of the larger divided by the smaller, and we want the opposite, so we get $r = 1/1.2 = 5/6$. This is the ratio by which you have to multiply each volume to get the next smaller volume. The total volume, then, is:

$$9 + 9(\frac{5}{6}) + 9(\frac{5}{6})^2 + 9(\frac{5}{6})^3 \dots.$$

This geometric series sums to $\frac{9}{1-\frac{5}{6}} = 54$. So the total enclosed volume is 54 cubic inches.

By the way, the numbers given in this problem are not simply made up, but are deduced from the size and shape of a large adult chambered nautilus. The number 54 is the approximate volume of a cylinder with height 2 inches and radius 3 inches (a rough approximation to the organism's size and shape).

Where does $5/6$ come from? Notice that one "band" of the chambers takes about 17 chambers, and (by directly measuring the picture), shrinks the organism by a factor of 3, *in length*. Scaling down by a factor of 3 in length is the same as scaling by a factor of 27 in volume, which should leave $54/27 = 2$ cubic inches. Therefore the first 17 chambers take 52 cubic inches. So we have the equations:

$$\frac{a}{1-r} = 54 \text{ and } \frac{a(1-r^{17})}{1-r} = 52.$$

Solving simultaneously gives $a = 9.5 \approx 9$, $r = .82 \approx 5/6$. This is how the problem was written.